Chapter 5.
Active Ranging Sensors

5.1. Overview

The sensors operational principles are the same for electromagnetic (radar, laser etc.) and active acoustic sensing.

Source of radiation is fed to a transmit antenna
- Tuned to the characteristics of the target (sometimes)
- Matched to the impedance of the medium to maximise coupling and efficiency
- Radiated by a directional antenna to increase the energy on target (sometimes)

Impacts on the target of interest
- Change in impedance results in re-radiation or scattering
- Re-radiation isotropic, random or directional

A small % of the power enters the receiver antenna
- Converted to an electrical signal
- Amplified and detected
5.2. Principle of Operation

![Figure 5.2: Operational principles of a pulsed time-of-flight radar illustrate that the echoes from the nearer houses return to the radar first and that the echoes from the last two houses overlap and appear to the radar as a single return](image)

Time-of-flight is the principle mode of operation for most radar, laser and active acoustic devices. This technique uses the time between the transmission of a pulse and the reception of an echo to provide range.

Because the round-trip time is measured there is a factor of 2 in the formula as shown below,

\[ R = \frac{v\Delta T}{2}, \]  

(5.1)

where \( R \) – range (m),
\( v \) – wave propagation velocity (m/s),
\( \Delta T \) – round trip time (s).

5.2.1. Requirements

To operate efficiently a narrow beam must be formed to concentrate the transmitted energy, the transducer must be matched to the characteristics of the medium and the receiver must match the transmitter characteristics.
Some form of modulation must “mark” the carrier signal so that the round trip time can be measured. The figure below shows an amplitude modulated carrier showing the “bang” pulse and then a pair of echoes after a short delay.

![Figure 5.3: Actual received signal showing carrier modulation](image)

It is not easy to process the modulated carrier, so in general it is detected (demodulated) before the timing information can be extracted. The figure below shows the envelope detected output.

![Figure 5.4: Received signal after envelope detection](image)

5.2.2. **Speed of Propagation for EM Radiation**

Propagation speed is a function of the refractive index of the material

\[ v_{\text{mat}} = \frac{c}{N} = \frac{c}{\sqrt{\varepsilon_r}} \text{ m/s}, \]  

(5.2)

where  
- \( c \) – Speed of Light in a Vacuum (3×10^8 m/s),  
- \( N \) – refractive Index of the material,  
- \( \varepsilon_r \) – Relative dielectric constant of the material.

5.2.3. **Speed of Propagation of Sound Waves**

For acoustic propagation the velocity is a function of the bulk modulus and the density for sound waves as calculated below

\[ v_{\text{mat}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{1}{K\rho}} \text{ m/s}, \]  

(5.3)

where  
- \( B \) – Bulk Modulus of the material,  
- \( K \) – Compressibility of the material,  
- \( \rho \) – Density of the material (kg/m^3).
The bulk modulus of the material is defined as ratio of the applied pressure to the strain \( B = \frac{P}{(V_o - V_n)/V_o} \) Where \( P \) is the pressure and \( V \) is the volume.

The bulk, \( B \), modulus of air is equal to 1.4 times the pressure. At sea level \( P_{air} = 1.01 \times 10^5 \text{ N/m}^2 \) and \( \rho_{air} = 1.20\text{kg/m}^3 \).

<table>
<thead>
<tr>
<th>Material</th>
<th>Speed (m/s)</th>
<th>Material</th>
<th>Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air (STP)</td>
<td>343.2</td>
<td>Sea water</td>
<td>1450 - 1750</td>
</tr>
<tr>
<td>Air 0°C, dry</td>
<td>331.29</td>
<td>Methanol 30°C</td>
<td>1121.2</td>
</tr>
<tr>
<td>Nitrogen 0°C</td>
<td>334</td>
<td>Mercury</td>
<td>1451</td>
</tr>
<tr>
<td>Oxygen 0°C</td>
<td>316</td>
<td>Aluminium</td>
<td>5000</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>259</td>
<td>Copper</td>
<td>3750</td>
</tr>
<tr>
<td>Helium 0°C</td>
<td>965</td>
<td>Lead</td>
<td>1210</td>
</tr>
<tr>
<td>Water 0°C</td>
<td>1402.3</td>
<td>Steel</td>
<td>5250</td>
</tr>
</tbody>
</table>

Note that the temperature, the salinity and the density determine the variation in the speed of sound in the sea. This is addressed in Chapter 8. For time of flight sonar depth sounders, this variation in speed must be considered if accurate measurements are required.

### 5.2.4. The Antenna

The antenna acts as a transducer between transmission line propagation and free-space propagation.

During transmission it concentrates the radiated energy into a shaped beam that points in the desired direction and only illuminates the selected target and on reception it collects the energy reflected by the target and delivers it to the receiver.

The effectiveness of these two roles is described by the transmitting gain and the effective receiving aperture of the antenna.

In Sonar applications, the gain is known as the directivity index

Either one or two antennas can be used.

The gain and beamwidth are determined by the size of the antenna in terms of the wavelength radiated,

\[
G = \frac{4\pi A}{\lambda^2},
\]

where: \( G \) – Antenna gain,
\( A \) – Frontal area of the antenna (m^2),
\( \lambda \) – Wavelength (m).
The antenna beamwidth is inversely proportional to the gain, so it can be estimated using the following simple formula.

\[
G \approx \frac{4\pi}{\theta_{3dB} \phi_{3dB}},
\]

where \( \theta_{3dB} \) and \( \phi_{3dB} \) are the beamwidths in orthogonal planes (rad).

\[
\theta_{3dB} \approx \frac{70\lambda}{d} \text{ deg},
\]

where \( \lambda \) is the wavelength (m) and \( d \) the antenna diameter (m).

The power pattern in the far field is proportional to the square of the field magnitude. \( |F(u)|^2 \) for both sonar and radar antennas.

For a circular aperture the power pattern is

\[
P = \left| \frac{J_1[(\pi D / \lambda) \sin \theta]}{\pi D / \lambda \sin \theta} \right|^2,
\]

where \( J_1 \) is the Bessel function of the first kind.

The half power beamwidth is \( \theta_{3dB} = \sin^{-1}(1.029\lambda / D) \) which corresponds (in the following figure) to \( u = 1.616 \).

For a rectangular aperture the transmitted (or received) power pattern as a function of the angle off boresight is

\[
P = \left| \frac{\sin[(\pi l / \lambda) \sin \theta]}{(\pi l / \lambda) \sin \theta} \right|^2.
\]

The half power beamwidth is \( \theta_{3dB} = \sin^{-1}(0.887\lambda / l) \) which corresponds to \( u = 1.393 \).
Figure 5.5: Comparison between far field patterns of circular and rectangular antenna apertures

For the Sonar application, the electric field intensity is replaced by the sound pressure. In the following figure, the relationship between the aperture and the beamwidth is compared for three different transducer configurations.

Figure 5.6: Total beam angle for (A) ring diameter, \( D \), (B) line length, \( L \) and (C) piston diameter, \( D \) relative to the wavelength \( \lambda \).
Circular and Rectangular Antennas
Tracking and ranging antennas are generally symmetrical in the two axes so that they can generate a pencil beam pattern.

Cylindrical Antennas
Antennas and transducers for imaging or searching are generally asymmetrical so that they generate a broad beam in elevation and a narrow beam in azimuth.
5.2.5. The Transmitter

To mark the carrier so that the time-of-flight can be measured, the signal is “marked” in some way. This is known as modulation, and can be achieved in various ways

- Amplitude modulation (AM)
- Frequency modulation (FM)
- Phase modulation (PM)
- Polarisation modulation

For most simple TOF sensors, the carrier is on-off amplitude modulated as shown in the figure.

\[ \frac{v T_{\text{max}}}{2}, \quad (5.9) \]

where:
- \( R_{\text{max}} \) – Maximum unambiguous range (m),
- \( v \) – Velocity (m/s),
- \( T_{\text{max}} \) – Pulse repetition interval (1/PRF).

The pulse width \( \tau \) is selected to maximise the amount of energy transmitted (average power) while still giving the required range resolution.
It is possible to use “pulse compression” techniques to increase the average power while still maintaining the required range resolution. This is covered in Chapter 11.

**Radar Transmitters**

For long range applications, the peak transmit power can be extremely high, many MW for surveillance radars. Microwave tubes such as Magnetrons, Klystrons and Travelling Wave Tubes (TWTs) are generally used to produce the high powers required for radar operation.

The first Magnetron ever made is shown in the figure below. It was made by Randall and Boot of the Physics Department of Birmingham University in February 1940 and had important implications in the successful use of airborne radar during WW II.

![First Magnetron](image)

**Figure 5.11: The first magnetron made by Randall and Boot (without magnets)**

Magnetrons are still found in most low-cost radar systems because they are cheap and extremely efficient. The magnetron shown below is typical of the genre.

![Magnetron](image)

**Figure 5.12: Magnetron from (a) microwave oven and (b) 94GHz radar transmitter**

For short range applications at high frequency (>10GHz), solid state oscillators using Gunn or IMPATT diodes are often used. These are two-terminal devices which are placed within a resonant cavity where they can be biased to exhibit the appropriate behaviour to sustain high-frequency oscillations.
For operation at lower frequency, voltage controlled oscillators (VCOs) using FETs or high electron mobility transistors (HEMTs) are used. These use the more conventional amplifier and feedback configuration shown in Chapter 2 to produce oscillations.

For short range applications, most ultrasonic transmitters are made from a piezoelectric material. Such materials exhibit a change in size proportional to the applied electric potential.

Natural piezoelectric crystals include Quartz, Rochelle Salt (sodium potassium tartarate) and Tourmaline.
Ferroelectric crystals and ceramics of the barium titanate type and ferromagnetic ceramics of the ferrite type are also used as transducers. These materials exhibit changes in length proportional to the square (or higher power) of the polarisation, and exhibit changes in length which are a function of an applied electric or magnetic field. They are equivalent to piezoelectric and piezomagnetic materials.

Laser Transmitters

Laser transmitters rely on the quantum-mechanical operation of carefully doped PN junctions (diodes) to produce coherent radiation in the IR or visible region when biased by a short current pulse. As shown in the figure below, the cleaved ends of the diode function as mirrors to form a cavity within which the radiation is constrained as it is reflected back and forth.

The Receiver (Matched Filtering and Demodulation)

The receiver frequency response is matched to the transmitter modulation pulse characteristics to maximise the peak signal to noise ratio (SNR)

If the bandwidth of the receiver is wide compared to the bandwidth of the transmitted pulse, then noise introduced by the extra bandwidth reduces the SNR.
If the receiver bandwidth is too narrow, it will “ring” when it receives a pulse. As a rule of thumb for pulsed measurement systems, the receiver bandwidth $\beta$ is equal to the reciprocal of the pulse width.

$$\beta = \frac{1}{\tau}$$

(5.10)

The following figures show simulation results for an ultrasonic system in which the filter bandwidth is adjusted to illustrate this effect.

For other waveforms (pulse compression etc), the matched filter form must be calculated for the specific waveform and the type of noise. This is addressed in Chapter 11.

After filtering, the signal is demodulated (or detected) to extract the timing information from the carrier wave. For basic time-of-flight sensors, this is generally encoded in the amplitude of the carrier (AM).

In most laser sensors the reflected light is detected directly using a fast light-sensitive diode (PIN or avalanche photo diode).
5.3. Pulsed Range Measurement

The simplest method to determine the range to a target is to time the interval from the transmission of a pulse until the echo is received. As shown in the figure below this can be achieved by counting cycles from a high speed clock during this period. If the clock frequency is selected correctly for the propagation velocity, the counter output can be used to display the range directly.

![Figure 5.18: Measuring range using a time of flight sensor](image)

In this simple example, the echo signal envelope voltage is compared to a threshold that is adjusted so that the probability of a false alarm (noise alone exceeding the threshold) is low, while there is still a good probability that the signal + noise will exceed the threshold.

The threshold level can be adjusted automatically, or the receiver gain can be adjusted automatically to compensate for the $R^{-4}$ signal-level characteristic. This is called sensitivity time control (STC). It should not be confused with automatic gain control (AGC) which performs a different function.

![Figure 5.19: Adjustable detection threshold to accommodate propagation losses for targets with a constant cross section](image)

Because the velocity of electromagnetic radiation is so high, this method cannot easily be used to measure the target range to an accuracy of better than about 1m. However it is still possible to measure the range to a fraction of a metre using the split-gate technique described below.
5.3.1. How the Split Gate Estimator Operates

The split-gate estimator generates a linear correction voltage that is proportional to the time of arrival of the echo relative to a clock cycle. Depending on the signal to noise ratio, this correction can improve the accuracy of the range measurement from a few metres down to a few centimetres.

To illustrate the process, in this example it is assumed that the effective pulse width is equal to the $\pm 1\sigma$ limits of the pulse shown in the figure below. For $\sigma = 1$, this equates to an effective pulse width of 2 range units.

![Figure 5.20: Received pulse shape assumed to be Gaussian](image)

**Gating process**

- A clock with a period of 2 range units runs continuously. This is easy to achieve as a typical LIDAR generates pulses 20ns long. As these are equal to the clock period, its frequency will be 50MHz.
- A comparator with a set threshold starts a counter on the leading edge of the bang pulse. The counter counts on the rising edge of the clock pulse.
- A similar comparator stops the counter on the leading edge of the target echo pulse.
- This comparator also enables two identical fast sample and hold (S&H) circuits.
- These S&H circuits are triggered on the next rising edge of the clock. The first samples the direct echo pulse, and the second samples the same pulse that has passed through a delay (section of coax cable) equal to one half a range unit.
- The outputs of these two S&H circuits are combined in the standard normalised split-gate configuration to determine the offset of the pulses from the rising edge of the clock signal.
Range = K x Count(N) + J x ΔR + Offset

Figure 5.21: Split gate range estimator schematic

Figure 5.22: Transfer function of different magnitudes of gate separation
The error function must be linear over a single clock cycle (equal to two range units) as shown in Figure 5.23. In Figure 5.24, the linear portion of the transfer function is shown to extend further than that required.

The following MATLAB script determines the range error transfer function of a split-gate estimator.

```matlab
% split gate technique to measure target range of a laser range finder
% Laser01.m
% % generate the Gaussian pulse amplitude
% r=-4:0.01:4;
% v=exp(-r.*r/2)/(sqrt(2*pi));
% plot(r,v);
% grid
title('Gaussian Pulse: Sigma=1')
ylabel('Amplitude')
pause

% separate the pulses
% dr = 0.5;
% vdel = exp(-(r-dr).*(r-dr)/2)/(sqrt(2*pi));
% plot(r,v,r,vdel)
% grid
title('Delayed and Direct Pulses')
xlabel('Range Offset')
ylabel('Amplitude')
```
5.3.2. Pulse Integration

Pulse integration may be used to decrease the false-alarm rate and to improve the measurement accuracy by increasing the effective signal to noise ratio of the echo. This can be achieved by averaging a number of pulses before thresholding, or by averaging the measured range and discarding outliers.

![Figure 5.25: Pseudo pulse integration](image)

<table>
<thead>
<tr>
<th>Count</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>819</td>
<td>Outlier, Ignore</td>
</tr>
<tr>
<td>1165</td>
<td>Average</td>
</tr>
<tr>
<td>2047</td>
<td>No det, Ignore</td>
</tr>
<tr>
<td>1130</td>
<td>Average</td>
</tr>
<tr>
<td>1176</td>
<td>Average</td>
</tr>
</tbody>
</table>
Integration of pulses requires a storage device that allows the individual echo returns to be averaged. In the old days this was achieved by using a long persistence phosphor on the cathode ray tube radar display combined with the integrating properties of the eyes and brain of the operator. However, modern ranging devices often use digital memory and high-speed signal processing techniques to perform the integration function as shown in the figure below.

![Digitised Echo Amplitude Profile](image1)

**Figure 5.26: True pulse integration**

Digital processing of radar signals is limited by the sample rate of the required Analog to Digital (ADC) converters with sufficient dynamic range.

![Digital Pulse Integration Method](image2)

**Figure 5.27: Digital pulse integration method**

If the pulse width $\delta R = 3m$ then the aperture time for the Sample and Hold (S&H) should be

$$\tau = \frac{2\delta R}{c} = 20\text{ns},$$

(5.11)

and the ADC must be clocked at 50MHz to capture every sample. It is therefore not practical to use this integration technique for very short-range high-resolution techniques as the S&H aperture would have to be too short.
5.3.3. Time Transformation

Cost sensitive applications such as industrial level sensors and commercial laser range finders use an analogue technique known as “time transformation” to stretch the return time as this leads to a reduction of their clock frequency and ADC requirements. This technique relies on repeated sampling of many pulse cycles.

In this technique, two timing ramps are generated, a fast ramp with a period equal to the pulse repetition interval (PRI) and synchronised to it, and a slow ramp that may span thousands of pulses. A comparator triggers a fast S&H every time the fast-ramp voltage exceeds that of the slow ramp, and the sampled echo signal is held constant for just over one complete PRI cycle. From the figure below, it can be seen that this output tracks the echo sequence but transformed in time from the original PRI by the ratio of the slow-ramp period to that of the fast ramp.

![Figure 5.28: Integration by time transformation](image)

Typical transformation ratios of (1:100000) are common in short range level measurement applications as these will transform a time-of-flight sensor that uses EM radiation into the equivalent of an acoustic system.

It should be noted that this technique is only required for radar and laser devices because of the high speed of EM propagation. This is not required for sonar devices in air (or even in water) as the speed is sufficiently low.

Finally, time transformation has the added advantage of integrating repetitive signals automatically because each point on the repeated echo is sampled many times and integrated through a low-pass filter.

5.4. Other Methods to Measure Range

In theory it is possible to obtain extremely accurate ($\delta R < 1 \text{mm}$) measurements using standard TOF methods with the split gate process. However, in practise, the pulse widths need to be extremely short and hence the bandwidth must be very high, and this makes achieving the required signal to noise ratio extremely difficult. An alternative is to use the relative phase of the received signal to determine the range.
The original phase reference technique was developed by Dr. T.L. Wadley of the South African National Institute of Telecommunications in 1957 and implemented as a microwave Tellurometer MRA-1.

Phase reference measurements are slow and are only suitable for point targets, so if a faster update rate is required and multiple targets are present in the beam, then more conventional wideband modulation is used.

5.4.1. Ranging using an Unmodulated Carrier

The most basic ranging method involves measuring the phase shift of the unmodulated carrier at at least two different frequencies to determine the range as shown in the following figures.

The equation that defines the range to the target in terms of the wavelength of the signal is

\[ 2D = n\lambda + \Delta\lambda, \]

where
- \( D \) – Distance to the target (m),
- \( \lambda \) – Carrier wavelength (m),
- \( n \) – Number of whole wavelengths,
- \( \Delta \) - Fractions of wavelength.

Because it is difficult to measure phase shifts exactly, an alternative technique is to adjust the transmitted frequency so there is a whole number of cycles in 2D.

By shifting the frequency further until this occurs again, two equations can be written and solved for the distance to the target, \( D \),

\[ 2D = n_1\lambda_1 \]
\[ 2D = (n_1 + N)\lambda_2 \]

where
- \( n_1 \) – Unknown number of cycles in the round trip distance,
- \( \lambda_1 \) – Wavelength at carrier frequency \( f_1 \),
- \( N \) – Number of full cycles of phase shift,
- \( \lambda_2 \) – Wavelength at carrier frequency \( f_2 \).
Solving for $D$ in terms of the frequency

$$D = \frac{Nc}{2(f_2 - f_1)},$$  \hspace{1cm} (5.14)

where $c$ – Speed of light (m/s),
$f_2$ – Frequency 2 (Hz),
$f_1$ – Frequency 1 (Hz).

### 5.4.2. Ranging using a Modulated Carrier

Most systems use a master station and a remote transponder which amplifies and retransmits the signal to maximise the operating range, however, this is not required if a good reflecting surface is available. As before the round trip distance between the master and the transponder can be given in terms of wavelengths

$$2D = n\lambda_m + \Delta\lambda_m$$  \hspace{1cm} (5.15)

where $\lambda_m$ – Modulation signal wavelength (m)
$n$ – Number of whole wavelengths
$\Delta$ - Fractions of wavelength

With this equation, both $n$ and $D$ are unknown, only the phase difference $\Delta\lambda_m$ can be measured using the principles shown in the following figure.

---

**Figure 5.30:** FM and AM modulated carrier based CW range measurement techniques
A wavelength of 10m is a compromise between the ability to resolve phase differences (larger $\lambda$) and the ultimate resolution (smaller $\lambda$)

EM radiation with $\lambda = 10m$ equates to $f = 30MHz$ for which it is impractical to make directional antennas for portable operation. So, in general, a higher frequency carrier is used that is modulated with a 30MHz signal.

Resolving the integer ambiguity, $n$, can be achieved in a number of ways

Decade modulation involves stepping $\lambda$ multiples of 10, eg, 10km->1km->100m->10m->1m etc. The range must be known to within a few km. This technique is used by the CA1000 and MA100 Tellurometers.

This process was manual and time consuming as the results had to be tabulated and the range calculated for each measurement. It also requires the modulator frequency to be very accurate and stable.

Modern IR systems like the MA-200 use a series of similar, stable frequencies. The phase difference is measured for each frequency and the data used to solve a series of simultaneous equations.

**Phase Detection**

A new range of wideband phase detector ICs such as the AD8302 perform accurate phase comparisons over a wide range of frequencies from close to DC up to 2.7GHz and over a wide range of input powers (-60dBm to 0dBm)

![Figure 5.31: AD8302 phase detector performance at 100MHz and -30dBm](image)

The output is calibrated to 10mV/deg with a typical nonlinearity of $<1$deg.
5.4.3. Tellurometer Example

For example, if the one modulation was set to a 10m wavelength to resolve the distance to the nearest mm, the other would be set to 9.990m.

This would allow the phase to be resolved without ambiguity out to a range of 5km (2D < 10km)

\[
2D = n_1 \lambda_1 + \Delta \lambda_1 \\
2D = n_2 \lambda_2 + \Delta \lambda_2
\]

(5.16)

If \( \lambda_1 = 10m \) and \( \lambda_2 = 9.99m \) and the phase difference measurements are respectively 0.0363\( \lambda_1 \) and 0.5989\( \lambda_2 \), what is the measured range.

Because \( \Delta \lambda_2 > \Delta \lambda_1 \), \( n_1 = n_2 = n \), and the equations can be written as follows

\[
2D = 10n + 0.0363 \times 10 \quad [1] \\
2D = 9.99n + 0.5989 \times 9.99 \quad [2] \\
[2]-0.999[1] \\
2D(1-0.999) = 9.99n - 9.99n + 9.99(0.5989 - 0.0363) \\
2D = 5.62036/0.001 \\
2D = 5620.363m
\]

If \( \Delta \lambda_2 < \Delta \lambda_1 \) then the phase has wrapped by a complete cycle and \( n_2 = n_1 + 1 \) and the equations solved accordingly.

5.4.4. Tellurometer Systems

MRA-101 (1962)

- Long range operation up to 50km in master/slave mode
- Narrow beamwidth <6° from 330mm diameter antenna
- Measuring time about 1 minute
- Accuracy +/- (1.5cm + 3 ppm)
- Frequency range 10.05 to 10.45GHz produced using a Klystron oscillator
- Modulation frequency 7.5MHz

Figure 5.32: MRA-101 Tellurometer
CA-1000 (1970)

- Range 50m to 30km in master/slave mode
- Slant polarised horn antenna size selected for 10 or 30km operation
- Accuracy +/- (1.5cm + 3ppm)
- Frequency range 10.1 to 10.45GHz produced using a Gunn oscillator
- Will operate as a point to point communication device

Figure 5.33: CA-1000 Tellurometer

MA-100

- Short range (<3km) high accuracy measurements
- Transmitter 0.92μm GaAs diode
- Photodiode receiver
- Amplitude Modulation (AM) frequency 75MHz
- Accuracy at 3km +/-1cm
- Phase measurement performed digitally using a pulse counting system

Figure 5.34: MA-100 Tellurometer

MA-200

- Range 10cm to 1km using a single prism
- IR laser (780μm) based transmitter with a beamwidth of 1.7mrad
- Accuracy +/-0.5mm +0.5mm per km
- Measurement time 45s (1st reading) 25s (subsequent readings)
5.5. The Radar Range Equation

The radar range equation determines the maximum range at which a target can be detected. It takes on many forms depending on what type of system is being described and it can be used to describe Lasers, Radars or Acoustic Systems.

It is generally optimistic because of losses not accounted for and higher than expected noise levels reduce the performance of actual systems.

If the power transmitted by the sensor is $P_t$ through an isotropic radiator (one which radiates uniformly in all directions) then the power density at range $R$ from the transmitter will be equal to the transmitted power divided by the surface area $4\pi R^2$ of an imaginary sphere of radius $R$.

$$\text{Power density from an isotropic antenna} = \frac{P_t}{4\pi R^2} \text{ Watt/area} \quad (5.17)$$

The gain $G_t$ of an antenna is the measure of the increased power in the direction of the target compared to the isotropic case.

The power density $S_t$ at the target is

$$S_t = \frac{P_t G_t}{4\pi R^2} \text{ Watt/area}. \quad (5.18)$$

The target intercepts a portion of this power and re-radiates it in various directions.

The measure of the proportion of the incident power re-radiated in the direction of the receiver is denoted as the target backscatter coefficient $\rho$ for a laser radar with a spot size (footprint) smaller than the target, or as the radar cross-section $\sigma$ for either a laser or microwave radar if the beam footprint is larger than the target.
The radar cross section, RCS, has units of area and is characteristic of a particular target and its orientation.

The power density back at the receiver antenna will be

\[ S_R = \frac{P_t G_t \sigma}{4\pi R^2} \text{ Watt/area.} \]  
(5.19)

For an effective area of the receiving antenna \( A_e \) defined as the ratio of the received power at the antenna terminals to the power density of the incident wave. Then the power received by the sensor is

\[ S = \frac{P_t G_t \sigma}{(4\pi)^3 R^4} A_e \text{ W.} \]  
(5.20)

The gain \( G_r \) of a lossless antenna is related to its effective aperture by the expression

\[ G_r = \frac{4\pi A_e}{\lambda^2}, \]  
(5.21)

where \( \lambda \) is the wavelength (m).

The effective aperture of antennas that are large in terms of wavelength (\( d > 20\lambda \)) can approach the physical antenna size. Substituting for \( A_e \)

\[ S = \frac{P_t G_t G_r \lambda^2}{(4\pi)^3 R^4} \text{ W.} \]  
(5.22)

For a monostatic sensor (same receive and transmit antennas) \( G_t = G_r = G \)

\[ S = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \text{ W.} \]  
(5.23)

To account for losses, these are lumped together to reduce the amount of power received and are denoted \( L \) where \( L < 1 \)

\[ S = \frac{P_t G^2 \lambda^2 \sigma L}{(4\pi)^3 R^4} \text{ W.} \]  
(5.24)

The maximum detection range \( R_{\text{max}} \) is the range at which the received power just equals the minimum detectable signal \( S_{\text{min}} \). Equating \( R_{\text{max}} \) for \( S = S_{\text{min}} \).

\[ R_{\text{max}} = \left[ \frac{P_t G^2 \lambda^2 \sigma L}{(4\pi)^3 S_{\text{min}}} \right]^{1/2} \text{ metres} \]  
(5.25)
5.5.1. Example of Radar Detection Calculation

Determine the maximum detection range of a maritime surveillance radar with the following characteristics

Find the maximum detection range of a maritime surveillance radar with the following specifications
- Gain 34 dB
- Transmit peak power 2 kW \((10\log_{10}2000 = 33\text{dBW})\)
- Frequency 10 GHz
- Losses 5 dB
- Target RCS 5 sqm \((10\log_{10}5 = 7\text{dBsqm})\)
- Minimum detectable signal \(1.6 \times 10^{-13} \text{ W} \ (-128\text{dBW})\)

Start by converting everything to dB as shown

Apply the radar range equation that has been written for dB values

\[
S_{\text{dB}} = P_{\text{dB}} + 2G_{\text{dB}} + 10\log_{10}\left(\frac{\lambda^2}{(4\pi)^3}\right) + \sigma_{\text{dB}} - L_{\text{dB}} - 40\log_{10} R
\]  

(5.26)

Plot the received signal \(S_{\text{dB}}\) on a semilogx plot along with the minimum detectable power as shown.

![Graphical solution to the radar range equation](image)
5.5.2. Receiver Noise

Noise is the unwanted electromagnetic energy that interferes with the ability of the receiver to detect the wanted signal. It may enter the receiver through the antenna along with the desired signal or it may be generated within the receiver.

As discussed in an earlier lecture, noise is generated by the thermal motion of the conduction electrons in the ohmic portions of the receiver input stages. This is known as Thermal or Johnson Noise.

Noise power $P_N$ is expressed in terms of the temperature $T_o$ of a matched resistor at the input of the receiver

$$P_N = kT_o\beta \ W, \quad (5.27)$$

where: $k$ – Boltzmann’s Constant $(1.38\times10^{-23} \text{ J/K})$, $T_o$ – System Temperature (usually $290\text{K}$), $\beta$ – Receiver Noise Bandwidth (Hz).

The noise power in practical receivers is always greater than that which can be accounted for by thermal noise alone.

The total noise at the output of the receiver $N$ can be considered to be equal to the noise power output from an ideal receiver multiplied by a factor called the Noise Figure, $NF$.

$$N = P_NNF = kT_o\beta.NF \ W. \quad (5.28)$$

5.5.3. Detection Range in terms of Output SNR

Generally the target detection, $P_{d}$, and false alarm, $P_{fa}$, probabilities are given in terms of the output $\text{SNR}$.

The minimum received signal can be written in terms of the output $\text{SNR}$ required to achieve a specific $P_d$ and $P_{fa}$ and the pulse width $\tau$

$$\left(\frac{S}{N}\right)_{out} = \left(\frac{S_{min}}{N}\right)_{in} \beta\tau = \frac{S_{min}\beta\tau}{kT_o\beta.NF}, \quad (5.29)$$

$$S_{min} = \left(\frac{S}{N}\right)_{out} \frac{kT_o.NF}{\tau}. \quad (5.30)$$

Substituting $S_{min}$ back into the radar range equation to obtain the detection range

$$R_{max} = \left[ \frac{P_{d}G^2\lambda^2\sigma L\tau}{(4\pi)^3kT_o(S/N)} \right]^{\gamma/2} \text{metres.} \quad (5.31)$$
5.5.4. Pulse integration and the probability of detection

If all of the received power from $K$ radar returns is integrated perfectly, then the effective $SNR$ is equal to $K$ times the single pulse $SNR$ for coherent (pre-detection) integration.

Most radars perform their integration on the video signal after envelope detection as it is much easier to achieve. This is known as non-coherent integration. This is not as effective as coherent integration and the improvement in effective $SNR$ is only about $K^{0.8}$ for white thermal noise.

For correlated noise the effectiveness of integration is reduced and is a function of the correlation time and the number of pulses integrated (the integration time).

The following probability density function (PDF) shows the effect of integration on the noise and the target + noise distributions for a non-fluctuating target echo.

![Effect of integration on noise and signal PDF's](image)

Note that the means of the PDFs remain unchanged, but their separation (in terms of variance) increases.

For a given threshold, the probability of detection $P_d$ increases and the probability of false alarm $P_{fa}$ decreases. These probabilities are generally evaluated numerically and displayed graphically as shown in the figure below.

Generally, a $P_d$ and $P_{fa}$ are selected for a particular application. The required $SNR$ is then determined from the graph. This $SNR$ is then reduced by the effective integration gain $10\log_{10}(K)$ for coherent integration and $8\log_{10}(K)$ for non-coherent (also known as post-detection) integration where $K$ is the number of pulses integrated before being plugged into the range equation.

Note that any fluctuations in the target RCS has a major influence on the $P_d$ and $P_{fa}$ or on the detection range. These effects are dealt with in Chapter 10.
5.6. Issues with TOF Measurement

Electromagnetic waves travel very fast with $\delta T = 6.6\text{ns/m}$. This means that high data rates are possible but it is difficult to obtain measurement resolution much better than 1m. As discussed earlier in this chapter, for a single point target, techniques exist to achieve measurement accuracies of a few mm.

Acoustic signals travel much more slowly $\delta T = 6\text{ms/m}$ (in the air). Data rates are much lower but good resolution and accuracy ($<1\text{mm}$) is possible with appropriate temperature compensation if the medium is still.

5.6.1. Range Ambiguity

Echoes from large targets at long range may be received after the next pulse has been transmitted. This problem can be eliminated by transmitting at different pulse repetition frequencies.
5.6.2. Beamwidth Ambiguity

Any object within the beam (or the sidelobes) will return an echo and as the actual target range may vary considerably across the beamwidth, there may be significant range uncertainty.

Targets with large cross sections will dominate the return echo even if they are not in the centre of the beam.

Compared to the possible range resolution, angular resolution of typical sensors is poor. It is determined by the effective aperture (in wavelengths) of the radiating antenna, and so is determined to a large extent by the transmission frequency.

Synthetic aperture, monopulse or interferometry can also be used to improve the angular resolution using techniques that are discussed in Chapter 12.

5.7. Examples of Range Measurement Radar

5.7.1. Radar Altimeter

Under certain conditions the radar cross section of a target may be a function of range.

\[ A = \frac{\pi d^2}{4} = \frac{\pi (R \theta)^2}{4}. \]  (5.32)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image}
\caption{Variations in the target cross section for a radio altimeter}
\end{figure}

In the case of a radar altimeter (shown above), the cross section, \( \sigma \), is proportional to the area of the beam footprint on the ground.

For a beamwidth \( \theta \), the area will be

\[ A = \frac{\pi d^2}{4} = \frac{\pi (R \theta)^2}{4}. \]  (5.32)
The scale factor $\sigma^o$ is called the target reflectivity and is defined as the radar cross section per unit area. The RCS is found by taking the product of the reflectivity and the area as discussed in detail in Chapter 9.

$$\sigma = \sigma^o A = \frac{\pi(\theta R)^2 \sigma^o}{4}. \quad (5.33)$$

Substituting into the basic radar range equation reduces the received signal power to a function of $R^2$

$$S = \frac{P_i G^2 \lambda^2 \theta \sigma^o L}{4^4 \pi^2 R^2}. \quad (5.34)$$

Because typical radar cross sections are large, and ranges are short, altimeters can transmit low average powers (100-500mW), and if required operate using low probability of intercept (LPI) spread spectrum waveforms

![Figure 5.39: Measurements made of Lake Victoria comparing radar altimeter data with that of a water level gauge](image-url)

Figure 5.39: Measurements made of Lake Victoria comparing radar altimeter data with that of a water level gauge
5.7.2. Cruise Missiles and Standoff Weapons

The formal definition of a cruise missile or standoff weapon is a dispensable, pilotless, self-guided, continuously powered, air breathing warhead delivery vehicle that flies like a plane.

The Boeing AGM-86B/C air-launched cruise missile, shown in the figure above, is a long-range subsonic, 1500kg self-guided missile carried by a B-52 bomber at high and low altitudes. Armed with a nuclear warhead, it is designated ALCM. With a conventional warhead, it is designated CALCM. The missile electronically "sees" the terrain over which it flies and can travel more than 2500km to hit the target.

Standoff weapons generally operate over much shorter ranges, hundreds of kilometres, than cruise missiles that can operate over thousands of kilometres.

Unlike ballistic missiles, cruise missiles require continuous guidance since both the velocity and direction of flight can be unpredictably altered by local weather conditions or unexpected changes in the performance of the guidance system.

A ballistic missile is guided for the first 5 of twenty minutes that it takes to fly 5000km; a cruise missile that flies at a subsonic speed would require at least 6 hours of continuously guided flight to cover the same distance.

Guidance errors that integrate with time would thus be 100 times larger for a cruise missile than for a ballistic missile operating at comparable range.

To obtain the necessary location information accuracy, a cruise missile generally uses a combination of inertial guidance that can be augmented by GPS when it is available and some form of map stored in its memory that can be used to correlate its position.
The earliest cruise missile was the German V-1 “buzz bomb” of World War II. However it and the US Matador, Regulus and Snark, and the Russian Shaddock missiles had no way of correlating their positions after launch.

Modern cruise missiles generally use one of two correlation techniques

- Terrain contour matching (tercom)
- Area correlation

![Figure 5.41: The V-1 (Doodle Bug) the original standoff weapon](image)

**5.7.3. Terrain Contour Matching (Tercom)**

Patented in 1958 it relies on the simple fact that the altitude of the ground above sea level varies as a function of location.

A terrain altitude map with 2km×10km with a cell size of 100×100m would require only 2000 points. So even if 20 such maps were required, the memory requirements are miniscule.

As the missile flies into one of the mapped areas, altitude readings obtained from an on-board radio altimeter are compared with the stored data to determine the missile position to the accuracy of one cell. The autopilot is then instructed to make the appropriate correction.

Inferences about drifts in the inertial system and prevailing winds etc. can be made at each stage, so subsequent maps may be smaller and higher resolution, leading to pinpoint targeting.
Figure 5.42: Cruise missile map matching

5.8. References
