High Angular Resolution Techniques

- Phased Arrays
- Doppler Beam Sharpening
- Synthetic Aperture
Resolution of a Single Aperture

- For imaging systems (not null steering trackers), the angular resolution is limited by the beam divergence.
- Beam divergence $\theta_{3dB}$ (beamwidth) is a function of the wavelength and the aperture size

$$\theta_{3dB} = \frac{k\lambda}{d}$$

where
- $\theta_{3dB}$ – 3dB Beamwidth
- $k$ – Constant (70 for° and 1.22 for rad)
- $\lambda$ - Wavelength (m)
- $d$ – Aperture diameter (m)

- The for a weighted aperture, the cross range resolution, $\delta$, is the product of the beamwidth (rad) and range (m)

$$\delta = R \theta_{3dB} = \frac{1.22 R \lambda}{d}$$

Requirement for Phased Arrays

- The wavelength is fixed by atmospheric window, propagation effects or physical size constraints.
- It is difficult to make the antenna diameter, $d$, arbitrarily large to obtain the required angular resolution because of manufacturing limitations.
- Use two or more transceivers in an array to synthesise an effective linear aperture equal to the array baseline.
- Uses
  - Phased array radars
  - Phased array sonar (linear or 2D) underwater or in the air
  - Long baseline radio telescopes
Transmitter Beam Synthesis

- Individual elements radiate precisely in phase to produce wave crests that move forward in phase
- Interfere constructively to produce a strong narrow beam directed straight ahead

Receiver Beam Forming

- The power received by each element is the sum of the received power scattered by target P from all the transmit elements
- The elements outputs are summed via lines of equal length to give $E_a$
Phase Shift Between Elements

- For an incoming signal at an angle $\theta$ to the array
- The phase shift between adjacent elements is $\psi$ (rad) where $d$ (m) is the element spacing

\[ \psi = \frac{2\pi d}{\lambda} \sin{\theta} \text{ rad} \]

- The centre of the array is taken as the phase reference

![Diagram of Phase Shift Between Elements]

Two Point Array

\[ E_d = \sin(\omega t + \psi/2) + \sin(\omega t - \psi/2) \]
\[ E_d = \sin(\omega t) \cdot 2\cos(\psi/2) \]
\[ \sin A + \sin B = 2\sin[(A+B)/2]\cos[(A-B)/2] \]
\[ E_d = \sin(\omega t) \frac{\sin(2\psi/2)}{\sin(\psi/2)} \]
\[ \sin 2A = 2\sin A\cos A \]

Amplitude Factor

The field intensity pattern $E_d(\theta)$ for the antenna is the magnitude of the amplitude factor
Expanding to N Point Array

Four Point Array

\[ E_d = \sin(\omega t + 3\psi/2) + \sin(\omega t + \psi/2) + \sin(\omega t - \psi/2) + \sin(\omega t - 3\psi/2) \]
\[ E_d = \sin(\omega t)(2\cos(3\psi/2) + 2\cos(\psi/2)) \]
\[ E_d = \sin(\omega t) \cdot \frac{\sin(4\Psi / 2)}{\sin(\Psi / 2)} \]

N Point Array

\[ E_d = \sin(\omega t) \cdot \frac{\sin(N\Psi / 2)}{\sin(\Psi / 2)} \]

The Field Intensity Pattern

- Substituting for \( \psi = 2\pi d \sin \theta / \lambda \)
  \[ E_d = \sin(\omega t) \cdot \frac{\sin \left( \frac{N\pi d}{\lambda} \sin \theta \right)}{\sin \left( \frac{\pi d}{\lambda} \sin \theta \right)} \]

- The general expression for the field intensity pattern \( E_d(\theta) \) is
  \[ |E_d(\theta)| = \left| \frac{\sin \left( \frac{N\pi d}{\lambda} \sin \theta \right)}{\sin \left( \frac{\pi d}{\lambda} \sin \theta \right)} \right| \]

- Nulls where the numerator is zero \( \frac{N\pi d}{\lambda} \sin \theta = 0, +/\pi, +/-2\pi \) etc

- The denominator is zero at \( \frac{\pi d}{\lambda} \sin \theta = 0, +/-\pi, +/-2\pi \) etc

- Applying L’Hopital’s rule where \( E_d = 0/0 \) we find maxima each with value \( N \) where \( \sin \theta = +/-n\pi/d \)
The Field Intensity Pattern cont…

- The maximum where $\sin\theta = 0$ is called the Main lobe, all the other lobes are called Grating lobes.
- If $d/\lambda = 0.5$, the grating lobe does not appear for $n = +/-1$ in real space because $\sin\theta > 1$ which is not possible.
- If $d/\lambda = 1$ the grating lobes appear at +/-90°, however as most real radiating elements do not radiate much at $\theta = 90°$, the grating lobes are suppressed.
- For a non scanning array, the best element spacing is $d = \lambda$.
- For a scanned array the best spacing is $d < \lambda/2$.

Grating Lobes for $N=10$ and $d/\lambda = 1$
No Grating Lobes for $N=20$ and $d/\lambda = 0.5$

Radiation Pattern: Linear Array

- The radiation pattern is defined as the normalised square of the amplitude factor

$$ G_a(\theta) = \frac{|E_a|^2}{N^2} = \frac{\sin^2 \left( \frac{N \pi d}{\lambda} \sin \theta \right)}{N^2 \sin^2 \left( \frac{\pi d}{\lambda} \sin \theta \right)} $$

- For $Nd = L$ (the length of the array) and for $\sin \alpha = \alpha$ (small angle)

$$ G_a(\theta) \approx \frac{\sin^2 \left( \frac{\pi L}{\lambda} \sin \theta \right)}{\left( \frac{\pi L}{\lambda} \sin \theta \right)^2} $$
Linear Array cont...

- For \( d = \lambda/2 \), the half power beamwidth \( \theta_{3dB} \) (rad) is as follows
  \[
  \theta_{3dB} = \frac{1.78}{N} \text{ rad}
  \]
  The beamwidth is smaller than normal because the sidelobes are so high – see p115-116

- If \( N \) is sufficiently large, the antenna will be equivalent to a uniformly illuminated aperture, and the first sidelobe will be 13.2dB down.

- For directive elements, the antenna pattern is the product of the element factor \( G_e(\theta) \) and the array factor \( G_a(\theta) \).
  \[
  G(\theta) = G_e(\theta) G_a(\theta)
  \]

Arrays of Directive Elements

- Directive Element Pattern
- Isotropic Array Pattern
- Directive Element Array pattern
2D Rectangular Array

The radiation pattern may be approximated as the product of the patterns of the two planes that contain the principle axes of the antenna

\[ G(\theta, \phi) = G(\theta)G(\phi) \]

\[ G(\theta, \phi) = \frac{\sin^2\left(\frac{N\pi d}{\lambda}\sin\theta\right)\sin^2\left(\frac{M\pi d}{\lambda}\sin\phi\right)}{\sin^2\left(\frac{\pi d}{\lambda}\sin\theta\right)\sin^2\left(\frac{\pi d}{\lambda}\sin\phi\right)} \]
Antenna Gain

- For large arrays, the non-scanned antenna gain can be approximated by the gain of a uniformly illuminated aperture

\[ G_o = \frac{4\pi A}{\lambda^2} \]

- For a scanned array, the gain is reduced by the scan angle \( \theta_o \) because the projected aperture is reduced in size.

\[ G(\theta_o) = \frac{4\pi A \cos \theta_o}{\lambda^2} \]
Beam Steering

- If the same phase is applied to all the elements of the array, then the main beam will be broadside to the array and $\theta = 0$
- The direction of the main beam will be $\theta_o$ if the relative phase difference between elements is

$$\varphi = \frac{2\pi d}{\lambda} \sin \theta_o$$

Steered Radiation Pattern

- The radiation pattern is then

$$G_a(\theta) = \frac{|E_a|^2}{N^2} = \frac{\sin^2 \left\{ \frac{N\pi d}{\lambda} (\sin \theta - \sin \theta_o) \right\}}{N^2 \sin^2 \left\{ \frac{\pi d}{\lambda} (\sin \theta - \sin \theta_o) \right\}}$$

- Grating lobes will occur at

$$\frac{\pi d}{\lambda} (\sin \theta_g - \sin \theta_o) = \pm n\pi$$

- For a scan over +/-90°, the element spacing should be $d = \lambda/2$
- For a practical array that can scan over +/-60°, the spacing $d > 0.54\lambda$
Corrections to Improve Range Resolution

- Phase shift only
- Delay adjust only
- Delay adjust and phase shift

Phase shift only

Delay adjust only

Delay adjust and phase shift

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12
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Half Power Beamwidth

- The half power beamwidth of a scanned array can be approximated by the following formula (not valid near endfire):

\[
\theta_{3dB} \approx \frac{0.886 \lambda}{Nd\cos \theta_o} \ \text{rad}
\]

- Taper is generally used to reduce the sidelobe levels. Using cosine on a pedestal \( A_n = a_0 + 2a_1 \cos(2\pi n/N) \) where \( 0 < 2a_1 < a_0 \)

\[
\theta_{3dB} \approx \frac{0.886 \lambda}{Nd\cos \theta_o} \left\{ 1 + 0.636 \left[ \frac{2d_1}{a_0} \right]^2 \right\} \ \text{rad}
\]
Matching and Mutual Coupling

- The impedance of the array elements varies with the scan angle.
- Spurious lobes may appear due to the mismatch.
- This is a difficult problem to solve analytically, and is often determined experimentally by exciting a single element and terminating all of the surrounding ones.
- Coupling is proportional to $1/d$ for $d=\lambda/2$, so the pattern and impedance are drastically altered by surrounding elements. Generally the surrounding 5x5 or even 9x9 elements must be considered.
The Ericsson Erieye system uses an active phased array radar mounted in a two sided array geometry contained in a large beam shaped structure carried above the fuselage. The limitation of the two sided array is that it can only cover two 120 degree sectors abeam of the aircraft, leaving 60 degree blind sectors over the nose and tail of the aircraft.

Thinned Arrays

- Reducing the number of elements leaves the main lobe unaltered but degrades the sidelobe level.
- Thinning to 10% reduces the main lobe level down to 10%, but leaves the sidelobe level at 90%.
- If the removed elements are replaced by matched dummy elements, then the pattern remains unchanged, only the gain is decreased.
Advantages of Phased Arrays

- Inertialess rapid beam steering
- Multiple, independent beams
- Potential for large peak and average powers
- Control of radiation pattern
- Graceful degradation
- Convenient aperture shape
- Electronic beam stabilisation

Pave Paws Early Warning Radar

- 1792 radiating elements
- Each array scans +/-60°
- 30m diameter
- 6000km range for 10m² target

- Note the 1980s graphical interface
Sea Based X-Band Radar

- Tracking and discrimination for ground-based midcourse defence
- Displacement 50,000 tons
- Tall as a 24 story building
- X-Band phased array, 65% populated

Acoustic Phased Array: Paul Thompson

- 16 radiating elements
- 16 receiver elements
- 3D imaging from a single “ping”
A side-scan sonar antenna is a short (≈ 50λ) linear transducer array made of a piezo-electric material that is towed behind a ship.

The transducer is excited by a short (τ ≈ 3 μs) high voltage sinusoidal stimulus at a frequency close to the resonant frequency of the array which the array converts to vibrations and radiates into the water.

The operational frequency is generally between 50kHz and 500kHz with some short-range units operating up to 1MHz.

The same array is used to receive any echoes. These are then amplified and recorded to form an image. In modern systems, the signal is digitised in the tow-fish and transmitted to the surface for processing and display.

The operational range for low frequency units (f ≈ 100kHz) is about 500m, this decreases to 50m at a frequency of 1MHz due to the increasing attenuation of water with frequency.
Sound Attenuation in Water

Attenuation increases by a factor of 10 from 100kHz to 1MHz

Beam Pattern

- Because of its shape, the array produces a fan beam pattern
  - narrow azimuth beamwidth (typ. 0.75 to 1.5°) determined by the length of the array
  - wide elevation beamwidth (typ. 35 to 65°) determined by the vertical aperture of each element.
- Arrays are placed on either side of the tow-fish and angled slightly downward to produce the patterns shown
Sidescan Image: The Port Hunter

Because of the shallow grazing angles, shadows can add significantly to the information available from a sidescan image.

Effect of Shadowing
Signal Processing

- The standard matched-filtering principles developed for radar are applied to sonar systems to ensure that the maximum SNR is achieved.
- Most side-scan systems are real beam in that their cross range resolution is a function of range $x_r = R \cdot \theta_{az}$.
- Digital techniques can be applied to correct for phase front curvature. This is known as focussing, and it can be used to achieve a fairly constant linear beamwidth with range (this is similar to SAR processing).
- Beam scanning techniques using phase shifters in the arrays can be used to spotlight particular areas.
- Simultaneous multi-frequency operation eg. 100kHz and 600kHz is possible for high resolution short range operation and lower resolution long range operation.

Pseudo 3D Images

If views are made from more than one perspective, they can be combined into a pseudo 3D image as shown here for the Fritzen.
Synthetic Aperture Radar

Definition

- Synthetic Aperture Radar (SAR) and Doppler beam-sharpening (DBS) are techniques that use the forward motion of an aircraft carrying a radar to improve the cross-range resolution.
- Both these techniques can be used for sonar applications as well.

Space Based SAR
Doppler Beam Sharpening

- Doppler beam-sharpening uses the decreasing radial velocity (hence Doppler shift) across the beam footprint to synthesise improved cross range resolution.
- For an aircraft flying at 250m/s, the isovel (and isodop) lines at 1.25m/s spacing are shown.

<table>
<thead>
<tr>
<th>Decrease in Velocity (m/s)</th>
<th>Azimuth Angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>5.73</td>
</tr>
<tr>
<td>2.5</td>
<td>8.11</td>
</tr>
<tr>
<td>3.75</td>
<td>9.93</td>
</tr>
<tr>
<td>5</td>
<td>11.48</td>
</tr>
<tr>
<td>6.25</td>
<td>12.84</td>
</tr>
<tr>
<td>7.5</td>
<td>14.07</td>
</tr>
</tbody>
</table>

Limitations include a trade off between the “sharpening” and the observation time.
- At 10GHz, isodop lines are 83Hz apart requiring an observation time of 12ms to resolve them, 3m travel time at 250m/s.
- At 94GHz, the lines are 800Hz apart requiring an observation time of 1.2ms (0.3m).
- The beam is scanned physically to the one side of the direction of travel.
- A reflectivity image is built up using the higher cross-range resolution.

Real Image

Sharpened Image
Developing a Synthetic Aperture

The term "synthetic aperture" refers to the distance that the sensor travels during the time that the reflectivity data are collected from a single point.

Energy from each point is made to arrive in phase at the output of the processor for all of the samples to realise the narrow beamwidth.

Good range resolution is obtained using one of the pulse compression techniques discussed in the previous lecture.

Generation of the Synthetic Aperture

Point target signature from a moving target, before and after pulse compression
The process to determine the radiation pattern is similar to that used for the fixed array.

The primary difference is that the signal received by each element is due only to the received power scattered by target P from one transmitter element.

This results in a slightly different radiation pattern for SAR. The beamwidth is narrower, but the sidelobes are higher than that for the equivalent phased array. For $L_s$, the synthetic array length:

$$G_{sar}(\theta) = \frac{\sin^2 \left( \frac{\pi L_s}{\lambda} \sin \theta \right)}{\left( \frac{\pi L_s}{\lambda} \sin \theta \right)^2}$$

The half power beamwidth can be found by solving for $G_{sar}(\theta)=0.5$ and solving graphically (or using Newton):

$$\frac{\pi L_s}{\lambda} \sin \theta \approx \frac{0.886\pi}{2} = 1.39 \text{ rad}$$
Unfocussed SAR

- Aircraft motion that deviates from a straight line and “range walk” is compensated for.
- One limiting condition for the largest aperture $L_{max}$ is the point where the phase error reaches $\frac{\lambda}{4}$ as determined in the diagram:

\[
\frac{\lambda}{4} = \frac{L_{max}}{\theta / 2} \sin(\theta / 2)
\]

\[
\sin(\theta / 2) \approx \frac{L_{max}}{2R}
\]

for

\[
\frac{\lambda}{4} \approx \frac{L_{max}^2}{4R}
\]

\[L_{max} = \sqrt{\frac{R\lambda}{\theta}}\]

Cross Range Resolution: Unfocussed SAR

- A second limiting condition is that the beamwidth is sufficiently wide to illuminate the target at point $P$. $L_{max} \leq R\theta_{3dB}$
- The beamwidth is obtained by equating $G_{SAR}(\theta)=0.5$ as before:

\[
\frac{\pi L_e}{\lambda} \sin \theta \approx \frac{0.886\pi}{2} = 1.39
\]

\[
\sin \theta = \frac{0.886\pi\lambda}{2\pi L_e}
\]

- The cross-range resolution $\delta_{cr} = R\theta = R\sin \theta$ for small angles:

\[
\delta_{cr} = \frac{R0.886\pi\lambda}{2\pi L_e}
\]

- Substituting for $L_e$ and simplifying $L_e = L_{max} = \sqrt{R\lambda}$:

\[
\delta_{cr} \approx 0.5\sqrt{R\lambda}
\]
**Focussed SAR**

Removal of the range curvature from the returns from a point target into a single range gate to allow correlation in azimuth that results in the improved cross-range resolution.

**A Doppler Perspective**

- A point scatterer enters the forward edge of the beam. It will have Doppler frequency:
  \[ f_d = \frac{2v_r}{\lambda} = \frac{2v}{\lambda} \cos(\theta_\text{end} / 2) \]
- For small beamwidths, the Doppler frequency decreases linearly to 0 and then increases again.
- The angle to the target as a function of time is:
  \[ \theta \approx \frac{vt}{R} \]
- The Doppler frequency as a function of time will then be:
  \[ f_d(t) = \frac{2v_r}{\lambda} = \frac{2v}{\lambda} \cos \left( \frac{vt}{R} \right) \]
Doppler Perspective cont…

- Taking the derivative to obtain the rate of change of Doppler frequency, or the Doppler slope

\[
\frac{df_d}{dt} = \frac{2v}{\lambda} \sin \left( \frac{\nu t}{R} \right)
\]

- At \( t=0 \)

\[
\frac{df_d}{dt} = \frac{2v^2}{R \lambda}
\]

- The total Doppler shift over time \( T_d = \) time within the beam for \( \theta = -\theta_{3\text{dB}}/2 \) to \( +\theta_{3\text{dB}}/2 \), assuming a linear change in frequency

\[
\Delta f_d = \frac{2v^2}{R \lambda} T_d
\]

- By analogy to the linear FM range resolution, the signal can be passed through a matched filter to give a spectral resolution

\[
\delta f = \frac{1}{T_d}
\]

Doppler Perspective cont…

- The cross range resolution is then the optimised cross range resolution of the real beam \( \delta_b = L_e \) scaled by the ratio of the spectral resolution to the whole Doppler shift

\[
\delta_{cr} = \delta_b \cdot \frac{\delta f}{\Delta f_d} = L_e \cdot \frac{\delta f}{\Delta f_d}
\]

Substituting

\[
\delta_{cr} = L_e \cdot \frac{R \lambda}{2v^2 T_d} \cdot \frac{1}{T_d} = L_e \cdot \frac{R \lambda}{2v^2 T_d^2}
\]

But \( L_e = v T_d = R \theta_b = R \lambda / D \) where \( D = \) antenna aperture

\[
\delta_{cr} = \frac{R \lambda}{2L_e} = \frac{R \lambda}{2} \cdot \frac{D}{R \lambda} = \frac{D}{2}
\]

The cross range resolution for focused SAR is independent of the range \( R \).
Resolution Comparison

Frequency $f = 77\text{GHz}$
Aperture $D = 150\text{mm}$

Perspective in radar imagery is somewhat different from the perspective in ordinary aerial photographs made with visible light. In the radar imagery two objects closer together than the imaging beam width will appear to coincide if they are both at the same range. The reason is that microwave pulses will reach them both at the same time and will return to the antenna at the same time, and so both objects will be seen as one. In photography two objects appear to coincide if they have the same angular coordinates as they are seen from the lens. Thus the perspective in the radar imagery is roughly analogous to perspective of a camera in the same vertical plane with the aircraft and objects but photographing the terrain at right angles.
Distortion in SAR Images

- layover, when the range to the top of an object is less than the distance to its base
- foreshortening, when the near side of elevated objects appears steeper than it actually is
- shadowing, when a tall opaque object blocks the signal path behind it, and no returns are received

Distortion in SAR Images: Stretching

- Deposition angle
- Ground range image plane
- Slant range image plane
- Slant Range Image
- Ground Range Image
Measurement Coherence: Speckle

Constructive Interference

Destructive Interference

Example of Homogeneous Target
(including image by a radar sensor)

Varying degrees of interference (constructive and destructive)

Raw Image

Filtered Image

China Lake Airfield 3m
Another Airport that isn’t Hong Kong

Piers and a River 1m
Pipeline Crossing a River 1m

T-72 Tanks in Formation 10cm
Space Based SAR

- To achieve good angular resolutions from real aperture space-borne radars is impossible at lower frequencies because the size of the antenna becomes prohibitively large.
- With a SAR, the large synthetic aperture results in a cross range resolution independent of range \( \delta_r = D/2 \) where \( D \) is the antenna aperture.
- The good range resolution \( \delta_r = c/2 \Delta f \) is achieved by transmitting a wide bandwidth chirp.
- Because the trajectory of the satellite or shuttle is precisely known and stable, motion compensation is not required and exceptionally high quality images can be produced.

Interferometry

- Because SAR is concerned with the phase relationships between scatterers on the ground:
  - Two similar images are produced using offset antennas, or on subsequent passes over the same area,
  - Interference patterns can be used to determine the true height of the objects on the ground.
- In addition to being useful for mapping ground features, this technology has a number of other uses:
  - Local deformation of the earth's crust as an early warning of earthquakes or volcanoes.
  - Ground subsidence due to mining activities or excessive use of ground-water.
Interferometric SAR Image of the San Francisco Area

Finally, earth in three dimensions

CAPE CANAVERAL: The Earth is mapping mission of the US shuttle Endeavour

The use of radar in business

A radar antenna mounted to the payload hatch sends very high frequency signals

The resulting “interferometric” image

The resulting “interferometric” image
Mississippi Delta

- Oil rigs
- Ship
Sif Mons 2km High and 300km in diameter
3D Image produced by combining SAR and altimeter data
Phased Array Application

Performance of Sidescan Sonar System

Sidescan System Evaluation
ITC 5202 Transducer

<table>
<thead>
<tr>
<th>Specification</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size of Unit</td>
<td>68.5x3.8cm</td>
</tr>
<tr>
<td>Array Dimensions</td>
<td>1.27x53cm shaded active array</td>
</tr>
<tr>
<td>Resonance Frequency</td>
<td>117kHz</td>
</tr>
<tr>
<td>Useable frequency range</td>
<td>111-126kHz</td>
</tr>
<tr>
<td>Beam pattern: 53cm line</td>
<td>1.5° at 117kHz</td>
</tr>
<tr>
<td>Beam pattern: 1.27cm line</td>
<td>60° at 117kHz</td>
</tr>
<tr>
<td>Efficiency</td>
<td>&gt;40%</td>
</tr>
<tr>
<td>Input power &lt;5% duty cycle</td>
<td>1500W</td>
</tr>
<tr>
<td>Operating Depth</td>
<td>Unlimited</td>
</tr>
<tr>
<td>Weight</td>
<td>4.3kg</td>
</tr>
<tr>
<td>Housing</td>
<td>aluminium</td>
</tr>
</tbody>
</table>
Transducer Array

Receive
-180dB rel 1V/μPa

Transmit
170dB rel 1μPa/V at 1m

Worked Example

- What is the smallest target that can be detected by the ITC-5202 transducer at a range of 500m?

- Operational frequency \( f = 117\text{kHz} \)
- Velocity of sound (assumed constant) \( c = 1522\text{m/s} \)
- Wavelength \( \lambda = c/f = 13\text{mm} \)
Pulse Width and Range Resolution

- The quality factor is determined from the operational band

\[ Q = \frac{f_r}{f_u - f_l} = \frac{117}{126 - 111} = 7.8 \]

- The rise time of any pulse generated by the transducer is related to the resonant frequency and the quality factor

\[ \tau_{\text{rise}} = \frac{Q}{f_r} = \frac{7.8}{117 \times 10^3} = 66.7 \mu s \]

- The minimum pulse-width must be at least twice the rise time if the pulse is to reach its peak value. For a "rectangular" pulse, the total pulse-width should be \( 5\tau_{\text{rise}} = 333\mu s \). This equates to a pulse width, and an effective range resolution of

\[ \delta R = \frac{c \tau}{2} = \frac{1522 \times 333 \times 10^{-6}}{2} = 0.25 m \]

---

Cross Range Resolution

- The cross range resolution at 500m (for the given azimuth beamwidth of 1.5°) is the product of the beamwidth in radians and the range (no focussing)

\[ \delta XR = R \theta = 500 \times \frac{1.5}{57.3} = 13.1 m \]
Pulse Repetition Frequency (PRF) and Duty Cycle (DC)

- To operate out to a maximum unambiguous range of 500m, the maximum pulse repetition frequency PRF is

\[ PRF_{\text{max}} = \frac{c}{2R_{\text{max}}} = \frac{1522}{2 \times 500} = 1.52 \text{Hz} \]

- The transmitter power is limited to a maximum of 1500W for a Duty Cycle of less than 5%. The duty cycle in this case is

\[ \text{DutyCycle} = 100 \times PRF \times \tau = 100 \times 1.52 \times 333 \times 10^{-6} = 0.05\% \]

- which is much smaller than the limit, so the maximum power can be applied to the transmitter.

Transmitter Power Density

- If the transducer was omnidirectional, then the power density at a range of 1m would be the product of the electrical power \( P_{\text{elec}} \), and the conversion efficiency \( \eta \) divided by the surface area of a sphere with a radius of 1m

\[ I_{\text{iso}} = \frac{P_{\text{elec}} \eta}{4\pi} = \frac{1500 \times 0.4}{4\pi} = 47.7 \text{W/m}^2 \]

- The antenna gain, known as the Directivity Index (DI), which is defined in terms of the power with respect to an isotropic radiator can be calculated from the elevation and azimuth beam widths (in radians)

\[ G = \frac{4\pi}{\theta \phi} = \frac{4\pi \times 57.3^2}{1.5 \times 60} = 458.4 \]

- The actual power density in the direction of the peak gain is the product of the gain and the isotropic value

\[ I = I_{\text{iso}} G = 47.7 \times 458.4 = 21864 \text{W/m}^2 \]
Sound Pressure Level

- The sound pressure level (SPL) or $S$ is generally given in dB relative to 1μPa at a range of 1m. This can be calculated from the power density and the acoustic impedance of the water.
- The acoustic impedance, $Z$ of water is the product of the density and the velocity

$$ Z = \rho_o c = 1026.4 \times 1522 = 1.56 \times 10^6 \text{ kg/m}^2 \text{ sec} $$

- The relationship between the acoustic pressure, $P$ in Pascals, the power density, $I$ in W/m$^2$, and the impedance, $Z$, is

$$ P^2 = IZ $$

- This can be re-written for the acoustic pressure in μPa for the sound pressure level, $S$, as follows

$$ S = (10^6 P)^2 = 10^{12} IZ $$

Sound Pressure Level in dB

- This is generally written in dB form

$$ 10\log_{10} S = 20\log_{10}(10^6 P) = 10\log_{10} 10^{12} + 10\log_{10}(IZ) $$

$$ S_{dB} = 120 + 10\log_{10}(21864 \times 1.56 \times 10^6) = 120 + 105.3 = 225.3 dB $$
Sound Pressure Level from Graphs

- It can be seen that at 117kHz, the transmitter voltage response is 170dB rel to 1μPa/Volt at 1meter.
- The electrical power input \( P_{\text{elec}} \) is related to the RMS voltage \( V \) and the transducer conductance \( G \)

\[
P_{\text{elec}} = V^2 G
\]

- For a conductance \( G = 5.5k \mu \text{mho} \) from the transducer specification table and a power \( P_{\text{elec}} = 1500W \)

\[
V = \sqrt{\frac{P_{\text{elec}}}{G}} = \sqrt{\frac{1500}{5.5 \times 10^{-3}}} = 522V_{\text{rms}}
\]

- The sound pressure level \( S_{\text{dB}} \) for 522V applied to the transducer is

\[
S_{\text{dB}} = 170 + 20\log_{10}(V) = 170 + 20\log_{10}(522) = 224.3dB
\]

Transmission Loss

- As the signal propagates through the water, the sound pressure level reduces because the wave is expanding on a spherical wavefront and due to attenuation. The transmission loss in dB is \( H \) and is determined as follows:

\[
H = 20\log_{10} \frac{r_2}{r_1} + \alpha_{\text{dB}} (r_2 - r_1)
\]

- Because the sound pressure is determined relative to the level existing at one meter from the effective centre of the sound source, the equation can be rewritten for this reference distance as follows

\[
H = 20\log_{10} r + \alpha_{\text{dB}} r
\]

- The attenuation in dB/m is given by the following formula where \( f \) is the frequency of the sound in kHz.

\[
\alpha_{\text{dB}} = \frac{0.036 f^2}{f^2 + 3600} + 3.2 \times 10^{-7} f^2 = \frac{0.036 \times 117^2}{117^2 + 3600} + 3.2 \times 10^{-7} \times 117^2 = 0.0329 \text{dB/m}
\]
**Target Strength \( T \)**

- \( T \) in dB is defined by ratio of the reflected sound pressure scattered by the target at a distance of one meter from the effective centre of the scattered sound to the incident sound pressure on the target
  
  \[
  T = 20 \log_{10} \frac{P_r}{P_i}
  \]

- This target strength is determined by its size, shape and the fraction of sound that is re-radiated.

- If the scattering cross section is \( \sigma \) square meters, then \( T \) in dB is given by the following formula
  
  \[
  T = 10 \log_{10} \frac{\sigma}{4\pi}
  \]

- As with the radar case, a sphere with a radius, \( a \), much larger than the wavelength will have a cross section equal to the projected area
  
  \[
  T = 10 \log_{10} \frac{\pi a^2}{4\pi} = 10 \log_{10} \left( \frac{a}{2} \right)^2 = 20 \log_{10} \frac{a}{2}
  \]

**Applying the Sonar Range Equation for a Spherical Target**

- For a spherical target, the echo sound pressure level \( E \) relative to 1\( \mu \)Pa at a range of 1m from the receiver is easily calculated as follows
  
  \[
  E_{dB} = S_{dB} - 2H + T
  \]

  \[
  E_{dB} = S_{dB} - 40 \log_{10} r - 2\alpha_{dB} r + 20 \log_{10} \frac{a}{2}
  \]
Range Eqn Applied to the Sea Floor

- Target strength will be the product of the range resolution and the cross range resolution modified by a scaling factor to take into account the reflectivity $\sigma^o$ of the surface.

$$ T = 10 \log_{10} \frac{\sigma^o A}{4\pi} = 10 \log_{10} \frac{\sigma^o \delta R \delta X R}{4\pi} = 10 \log_{10} \frac{\sigma^o \delta R \cdot \theta_{az}}{4\pi} $$

- and the echo sound pressure level 1m from the receiver will be

$$ E_{dB} = S_{dB} - 40 \log_{10} r - 2\alpha_{dB} r + 10 \log_{10} \frac{\sigma^o \delta R \cdot \theta_{az}}{4\pi} r $$

$$ E_{dB} = 225.3 - 40 \log_{10} r - 0.0656 r - 42.8 + 10 \log_{10} r $$

$S_{dB} = 225.3$dB
$\alpha_{dB} = 0.0328$ dB/m
$\sigma^o = 0.1$ m$^2$/m$^2$
$\theta_{az} = 1.5^\circ$ (0.0262 rad)
$\delta R = 0.25$m

Noise Floor

- The noise level at sea is mostly generated by wind and wave action on the surface. It is proportional to sea-state and inversely proportional to frequency.
- From the table reproduced in the notes, we will assume sea state 3 generated by a wind speed of 15 knots
- Isotropic Noise pressure $N_i$ (dB relative to $1\mu$Pa) into a 1Hz bandwidth at a frequency of 1kHz is 65dB
- The frequency relationship to map the noise pressure level at 1kHz to the transducer frequency is

$$ N_f = N_i - 17 \log_{10} f_{kHz} $$

- For a sea state 3 and the transducer frequency of 117kHz

$$ N_f = 65 - 17 \log_{10} 117 = 30$dB
Because the noise floor is defined for an isotropic receiver into a 1Hz bandwidth

The total noise pressure level in dB relative to 1μPa must take into account the bandwidth \( B \) of the transducer (in Hz) and its directivity or gain \( G \).

\[
L_N = N_f + 10 \log_{10} B - 10 \log_{10} G
\]

\[
L_N = 30 + 10 \log_{10} (15 \times 10^3) - 10 \log_{10} (458.4) \approx 45 dB
\]

Reverberation noise (the equivalent of volumetric clutter for a radar) will not be considered.
Targets Detectable at 500m

- The sea floor with a SNR = 25dB
- A sphere with a diameter of 1m with a SNR = 30dB
- A sphere with a diameter of 0.1m might be detectable but with an SNR = 10dB it cannot produce a $P_d = 0.9$ and a $P_{fa} = 10^{-6}$
The actual voltage output by the transducer is determined from the transducer specifications. The open circuit receiving response at 117kHz is –180dB re 1V/μPa. For a signal pressure of 70dB (the sea-floor return at 500m), the output is

\[
20 \log_{10}(V) = 70 - 180 = -110dB
\]

\[
V = 10^{-110/20} = 3.16\mu V
\]

This is very small, and so receiver noise would be a consideration when the actual detection characteristics of the system were being considered.