Chapter 12.

High Angular Resolution Techniques

12.1. Introduction

For imaging systems (not null steering trackers), the angular resolution is limited by the beam divergence. Beam divergence $\theta_{3dB}$ (beamwidth) is a function of the wavelength and the aperture size

$$\theta_{3dB} = \frac{k \lambda}{d},$$

(12.1)

where $\theta_{3dB}$ – 3dB Beamwidth,

- $k$ – Constant (70 for ° and 1.22 for rad),
- $\lambda$ - Wavelength (m),
- $d$ – Aperture diameter (m).

The cross range resolution $\delta_{XR}$ is the product of the beamwidth (rad) and range (m)

$$\delta_{XR} = R \theta_{3dB} = \frac{1.22 R \lambda}{d}$$ (m)

(12.2)

As the wavelength is often fixed by atmospheric window, propagation effects or physical size constraints and it is difficult to make the antenna diameter, $d$, arbitrarily large to obtain the required angular resolution because of manufacturing limitations, a solution is to use two or more transceivers in an array to synthesise an effective linear aperture equal to the array baseline.

Other techniques that use the forward motion of the antenna to synthesise a larger aperture (in one dimension only) include Doppler Beam Sharpening and Synthetic Aperture processing

12.2. Phased Array Synthesis

This technique is used by:
- Phased array radars
- Phased array sonar (linear or 2D) underwater or in the air
- Long baseline radio telescopes
The power received by each element is the sum of the received power scattered by target, $P$, from all the transmit elements. The outputs of all $N$ elements are summed via lines of equal length, without delays or phase shifts to give $E_a$.

Since each element observes phase $\phi_k$, the output after summing the signals from all $N$ elements directly, with no phasing is given by:

$$E_a = \sum_{k=1}^{N} \sin(\omega t + \phi_k)$$  \hspace{1cm} (12.3)

Figure 12.1: Phased array antennas (a) millimetre wave radio telescope (b) mortar locating radar (c) Jindalee over the horizon radar and (d) acoustic array for atmosphere profiling

Figure 12.2: The Array beam forming process is achieved by summing the signals from all the array elements in phase
Figure 12.3: Radiation from individual elements with a linearly increasing phase shift combines to form a linear phase-front at an angle to the array as shown by the crests of the waves.

For the example of a uniform phase front approaching the array at an angle \( \theta \), the relative phase shifts are 0, \( \psi \), 2\( \psi \), 3\( \psi \), …N\( \psi \) as shown in Figure 19.4.

![Figure 12.3](image)

Figure 12.4: Schematic diagram showing (a) the relative phase differences for a plane wave approaching a phased array and (b) construction used to determine the magnitude of the phase difference.

It is easy to see that the displacement, \( x \), corresponds to the phase shift between sequential elements and that it can be determined from the array spacing, \( d \), and the angle of arrival of the plane wave, \( \theta \)

\[
x = d \sin \theta
\]  

(12.4)

The phase shift, \( \psi \), that corresponds to the distance \( x \) is then

\[
\psi = \frac{2\pi x}{\lambda}
\]  

(12.5)

By substituting for \( x \) into (12.5)

\[
\psi = \frac{2\pi d}{\lambda} \sin \theta,
\]  

(12.6)

In general, however, the phase shifts are measured relative to the geometric centre of the array, and not element one. The geometric effect of this, is to displace the phase front represented by the line AB down to CD which passes through the geometric centre.
For the phase front AB, the relative phase shift of the elements starting at point B (k=1) is summed as follows

\[
E_a = \sum_{k=1}^{N} \sin[\omega t + (k-1)\psi]
\]  
(12.7)

For the phase front CD that passes through the geometric centre of the array, it can be seen that it is displaced by \((N-1)\psi/2\). The output can be re written

\[
E_a = \sum_{k=1}^{N} \sin[\omega t + (k-1)\psi - \frac{N-1}{2}\psi]
\]
\[
= \sum_{k=1}^{N} \sin[\omega t + k\psi - \frac{N+1}{2}\psi]
\]  
(12.8)

For the six element array, \(N = 6\), shown in Figure 19.4, the phase shifts become

Figure 12.6: Displaced phase shift across array

12.2.1. Two Point Array

Examining the elements on either side of the array centre shown in Figure 19.6, it can be seen that the phase shifts are \(-\psi/2\) and \(\psi/2\) and the sum will be

\[
E_a = \sin(\omega t + \psi/2) + \sin(\omega t - \psi/2)
\]  
(12.9)

Using the trig identity - \(2\sin A \cos B = \sin(A + B) + \sin(A - B)\)

\[
E_a = \sin(\omega t)2\cos(\psi/2)
\]  
(12.10)
From the trig identity - \( \sin 2A = 2 \sin A \cos A \), \( 2 \cos A = \sin 2A / \sin A \) therefore (12.10) can be rewritten as

\[
E_a = \sin(\omega t) \frac{\sin(2\psi / 2)}{\sin(\psi / 2)}
\]  
(12.11)

Figure 12.7: Field intensity pattern generated by two isotropic radiators

12.2.2. 4 Point Array

Once again, examining the elements on either side of the array in Figure 19.6, the sum will be

\[
E_a = \sin(\omega t + 3\psi / 2) + \sin(\omega t + \Psi / 2) + \sin(\omega t - \psi / 2) + \sin(\omega t - 3\psi / 2)
\]

(1) (2) (3) (4)

Grouping terms (1),(3) and (2),(4) and applying the trig identity \( -2 \sin A \cos B = \sin(A + B) + \sin(A - B) \)

\[
E_a = 2\sin(\omega t) \cos(3\psi / 2) + 2\sin(\omega t) \cos(\psi / 2)
\]  
(12.13)

Let \( A+B = \Psi / 2 \) and \( A-B = \psi / 2 \) and applying the same trig identity

\[
E_a = \sin(\omega t) 4\cos(2\psi / 2) \cos(\psi / 2)
\]  
(12.14)

Multiplying by \( \sin(2\psi / 2) / \sin(2\psi / 2) \), simplifying and applying the trig identity \( \sin 2A = 2 \sin A \cos A \)

\[
E_a = \sin(\omega t) \frac{2\sin(4\psi / 2) \cos(\psi / 2)}{\sin(2\psi / 2)}
\]  
(12.15)
Applying the trig identity again, and simplifying, the sum for a 4 point array is

\[ E_a = \sin(\omega t) \frac{\sin(4\psi / 2)}{\sin(\psi / 2)} \]  

(12.16)

12.2.3. The General Case

It can be seen that the only difference between the two element and the four element cases, is the size of the numerator. In the general case, for N elements, the numerator can therefore be rewritten as \( \sin(N\psi / 2) \) and the equation for the sum becomes

\[ E_a = \sum_{k=1}^{N} \sin \left( \omega t + k\psi - \frac{N+1}{2} \psi \right) = \sin(\omega t) \frac{\sin(N\psi / 2)}{\sin(\psi / 2)} \]  

(12.17)

Substituting for \( \psi \) from (12.6)

\[ E_a = \sin(\omega t) \frac{\sin \left( \frac{N\pi d}{\lambda} \sin \theta \right)}{\sin \left( \frac{\pi d}{\lambda} \sin \theta \right)} \]  

(12.18)

The second term of the equation is known as the amplitude factor of the array. The field intensity pattern, \( E_a(\theta) \), for the antenna is the magnitude of this amplitude factor.

The general expression for the field intensity pattern is therefore.

\[ |E_a(\theta)| = \sin \left( \frac{N\pi d}{\lambda} \sin \theta \right) \frac{\sin \left( \frac{\pi d}{\lambda} \sin \theta \right)}{\sin \left( \frac{\pi d}{\lambda} \sin \theta \right)} \]  

(12.19)

There are nulls where the numerator is zero: \( \frac{N\pi d}{\lambda} \sin \theta = 0, +/-\pi, +/-2\pi \) etc

The denominator is also zero at \( \frac{\pi d}{\lambda} \sin \theta = 0, +/-\pi, +/-2\pi \) etc

Applying L’Hopital’s rule where \( E_a = 0/0 \) it is found that \( E_a \) is a maximum where \( \sin \theta = +/-n\lambda/d \). These maxima will all have the value \( N \).

The maximum where \( \sin \theta = 0 \) is called the Main lobe, all the other lobes are called Grating lobes.

If \( d/\lambda = 0.5 \), the grating lobe does not appear for \( n = +/-1 \) in real space because \( \sin \theta > 1 \) which is not possible. If \( d/\lambda = 1 \) the grating lobes appear at \( +/-90^\circ \), and as most real radiating elements do not radiate much at \( \theta = 90^\circ \), the grating lobes are suppressed.
For a non scanning array, the best element spacing is $d = \lambda$ while for a scanned array the best spacing is $d < \lambda/2$.

![Figure 12.8: Ten element synthesis](image)

### 12.3. The Radiation Pattern

#### 12.3.1. Linear Array

The radiation pattern is defined as the normalised square of the amplitude factor

$$G_a(\theta) = \frac{|E_a|^2}{N^2} = \frac{\sin^2 \left( \frac{N \pi d}{\lambda} \sin \theta \right)}{N^2 \sin^2 \left( \frac{\pi d}{\lambda} \sin \theta \right)}. \quad (12.20)$$

For $Nd = L$ (the length of the antenna) and $\sin \alpha = \alpha$ (small angle aprox.)

$$G_a(\theta) \approx \frac{\sin^2 \left( \frac{\pi L}{\lambda} \sin \theta \right)}{\left( \frac{\pi L}{\lambda} \sin \theta \right)^2}. \quad (12.21)$$

For $d = \lambda/2$, then the half power beamwidth $\theta_{3dB}$ is as follows

$$\theta_{3dB} = \frac{1.73}{N} \text{ (rad).} \quad (12.22)$$

If $N$ is sufficiently large, the antenna will be equivalent to a uniformly illuminated aperture, and the first sidelobe will be 13.2dB down.
For directive elements, the antenna pattern is the product of the element factor $G_e(\theta)$ and the array factor $G_a(\theta)$,

$$ G(\theta) = G_e(\theta)G_a(\theta). $$  \hfill (12.23)

![Graphs of $G_a(\theta)$, $G_e(\theta)$, and $G(\theta)$](image)

**Figure 12.9**: Effect of directive elements on the antenna pattern where (a) is the array factor, (b) is the element factor (c) is the final antenna pattern

### 12.3.2. Radiation pattern: 2D Rectangular Array

The radiation pattern may be approximated as the product of the patterns of the two planes that contain the principle axes of the antenna

$$ G(\theta, \phi) = G(\theta)G(\phi), $$  \hfill (12.24)

$$ G(\theta, \phi) = \frac{\sin^2 \left( \frac{N\pi d}{\lambda} \sin \theta \right) \sin^2 \left( \frac{M\pi d}{\lambda} \sin \phi \right)}{\sin^2 \left( \frac{\pi d}{\lambda} \sin \theta \right) \sin^2 \left( \frac{\pi d}{\lambda} \sin \phi \right)}. $$  \hfill (12.25)
12.3.3. Antenna Gain

For large arrays, the non-scanned antenna gain can be approximated by the gain of a uniformly illuminated aperture

\[ G_n = \frac{4\pi A}{\lambda^2}. \]  

(12.26)

For a scanned array, the gain is reduced by the scan angle \( \theta_s \) because the projected aperture is reduced in size

\[ G(\theta_s) = \frac{4\pi A \cos \theta_s}{\lambda^2}. \]  

(12.27)
12.4. Beam Steering

Figure 12.11: Beam steering uses phase shifts to shift the region of constructive interference in the direction required. Note from (b) that the phase shift need never exceed one wavelength.

If the same phase is applied to all the elements of the array, then the main beam will be broadside to the array and \( \theta = 0 \).

The direction of the main beam will be \( \theta_o \) if the relative phase difference is

\[
\varphi = \frac{2\pi d}{\lambda} \sin \theta_o.
\]  

(12.28)

The phase of each element will be \( \varphi_c + m\varphi \), where \( m=1,2,3,\ldots \text{ N-1} \) and \( \varphi_c \) is a constant phase applied to all of the elements.

The radiation pattern is then

\[
G_o(\theta) = \left| \frac{E_o}{N^2} \right|^2 = \frac{\sin^2 \left( \frac{N\pi d}{\lambda} (\sin \theta - \sin \theta_o) \right)}{N^2 \sin^2 \left( \frac{\pi d}{\lambda} (\sin \theta - \sin \theta_o) \right)}.
\]

(12.29)

Grating lobes will occur at

\[
\frac{\pi d}{\lambda} (\sin \theta_g - \sin \theta_o) = \pm n\pi.
\]

(12.30)

For a scan over \(+/-90^\circ\), the element spacing should be \( d = \lambda/2 \), and for a practical array that can scan over \(+/-60^\circ\), the spacing \( d > 0.54\lambda \).
If the element spacing is not reduced, then as soon as the beam scans, the grating lobes will appear as shown in the following figure.

![Figure 12.12: Beam scanning for an element spacing $d = \lambda$ showing the appearance of a grating lobe](image)

12.4.1. Active and Passive Arrays

With the advent of low-cost MMICs it is now practical to manufacture individual transceiver modules to produce active arrays. Such arrays have been manufactured at X-band for less than $15 per module.

![Figure 12.13: Schematic diagram showing the configuration for (a) passive and (b) active arrays](image)

12.4.2. Corrections to Improve Range Resolution

Individual elements have to be fitted with phase delay circuitry to steer the beam. However, as the figure (a) below shows, if the squint angle is great, the pulse gets spread with a resultant decrease in the range resolution. To counter this, it is possible to use adjustable time delay circuits as shown in (b). However, in general, to minimise the amount of circuitry required in a system that might have thousands of elements, blocks of elements are delayed in time, before individual elements are steered using individual phase shifters.
Figure 12.14: Difference between Phase Delay and Time Delay. In (a) which uses only phase delays, the 5ns pulse is stretched out to 10ns. In (b) only time delays are implemented to maintain the 5ns pulse, and in (c) a compromise is implemented to minimise the expense of using individual time delay circuits for each element.

12.5. Array Characteristics

12.5.1. Half Power Beamwidth

The half power beamwidth of a scanned array can be approximated by the following formula (not valid near end-fire)

$$\theta_{3,db} \approx \frac{0.886 \lambda}{Nd \cos \theta_o} \text{ (rad).} \quad (12.31)$$

Generally some form of taper is used to reduce the sidelobe levels. Using cosine on a pedestal

$$A_n = a_0 + 2a_1 \cos (2\pi n/N)$$

where

$$0 < 2a_1 < a_0$$

$$\theta_{3,db} \approx \frac{0.886 \lambda}{Nd \cos \theta_o} \left(1 + 0.636 \left[\frac{2a_1}{a_0}\right]^2\right) \text{ (rad).} \quad (12.32)$$

12.5.2. Matching and Mutual Coupling

The impedance of the array elements varies with the scan angle and spurious lobes may appear due to the miss-match.

This is a difficult problem to solve analytically, and is often determined experimentally by exciting a single element and terminating all of the surrounding ones.
Coupling is proportional to \(1/d\) for \(d = \lambda/2\), so the pattern and impedance are drastically altered by surrounding elements. Generally the surrounding \(5 \times 5\) or even \(9 \times 9\) elements must be considered.

### 12.5.3. Thinned arrays

Reducing the number of elements leaves the main lobe unaltered but degrades the sidelobe level. In this example the 4000 element array is thinned to 900 elements. The mean sidelobe level is 31.5dB which is still acceptable.

Thinning to 10% reduces the main lobe level down to 10%, but leaves the sidelobe level at 90%.

If the removed elements are replaced by matched dummy elements, then the pattern remains unchanged, only the gain is decreased.

![Figure 12.15: Thinned array](image)

### 12.5.4. Conformal Arrays

It is theoretically possible to place array elements on any arbitrary surface and by applying the correct phase, amplitude and polarisation, to radiate a beam in the required direction:

- It is difficult to control the beam-shape and to get low sidelobes
- Feeding mechanism is difficult
- Each element in the array is different (unless there is symmetry) so impedance and coupling will be different
- Computationally difficult as there are no element or array factors.

Most of the work has been done on truncated cones, cylinders or ogives.

### 12.5.5. Advantages of using Phased Arrays

The following are the main advantages of using arrays:

- Inertialless rapid beam steering
- Multiple, independent beams
- Potential for large peak and average powers
- Control of radiation pattern
- Graceful degradation
- Convenient aperture shape
- Electronic beam stabilisation
12.6. Applications

12.6.1. Acoustic Array

The imaging acoustic array shown below was developed by Paul Thompson as part of his honours thesis.

![Acoustic Array Hardware](image1)

The following image shows a single sample synthesis made using an acoustic array made up from standard ultrasonic transducers.

![Acoustic Array Imaging](image2)

Figure 12.16: Acoustic array hardware

Figure 12.17: Imaging capability of acoustic phased array (a) photograph and (b) acoustic image (courtesy Paul Thompson)

12.6.2. Early Warning Radar Array

The figure below shows a photograph of Pave Paws, a long range early warning phased array radar that contains 1792 elements each radiating 322W peak power on a 30m wide face. It can detect targets with a 10m² cross-section at a range of 6000km.
The following photograph shows the sea based X-band (SBX) radar which has been developed for ground based midcourse defence. Its specifications are as follows:

- Built on a semi-submersible oil-drilling platform
- Deck area 68×58m
- Draft 19m
- Displacement 50,000 tons
- Sea surface to radome top 65m (24 stories)
- Radome 26m high and 30m in diameter, 7000kg
- Radar, X-band phased array, 65% populated
A number of studies have been conducted to determine whether the high levels of microwave radiation emitted by this and other large phased array radars are a health risk.

12.7. Phased Array Application: Side-Scan Sonar

12.7.1. Operational Principle

Similar in principle to side-looking airborne radar (SLAR), the side-scan sonar antenna is a short (≈50λ) linear transducer array made of a piezo-electric material that is towed behind a ship in a straight line.

The transducer is excited by a short (τ ≈ 3μs) high voltage sinusoidal stimulus at a frequency close to the resonant frequency of the array which the array converts to vibrations and radiates into the water.

The operational frequency is generally between 50kHz and 500kHz with some short-range units operating up to 1MHz.

Because of its shape, the array produces a fan beam pattern with a narrow azimuth beamwidth (typ. 0.75 to 1.5°) determined by the length of the array and a wide elevation beamwidth (typ. 35 to 65°) determined by the vertical aperture of each element.

The beam propagates through the water until it hits the bottom. The roughness of the floor, and any projections reflect sound waves back in the direction of the sonar.

The array is generally mounted within a streamlined body called a “tow-fish” that looks like a torpedo. A set of fins keeps the tow body in line with the tow track. Arrays are placed on either side of the tow-fish and angled slightly downward to produce symmetrical beams as shown in the following figure.
The same array is used to receive any echoes. These are then amplified and recorded to form an image. In modern systems, the signal is digitised in the tow-fish and transmitted to the surface for processing and display.

The operational range for low frequency units \( (f \approx 100\text{kHz}) \) is about 500m, this decreases to 50m at a frequency of 1MHz due to the increasing attenuation of water with frequency (see Chapter 8).

To minimise the geometric distortion, operation is restricted to shallow water, typically 20% of the maximum operating range.

### 12.7.2. Hardware

The specifications of a typical transducer that is mounted within the tow-fish is shown in the figure and table below
Table 12.1: Specifications of ITC-5202 side-scan array

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size of unit</td>
<td>68.5,\times,4,\times,3.8cm</td>
</tr>
<tr>
<td>Array dimensions</td>
<td>1.27,\times,53,cm shaded active array</td>
</tr>
<tr>
<td>Resonance frequency</td>
<td>117kHz</td>
</tr>
<tr>
<td>Usable frequency range</td>
<td>111-126kHz</td>
</tr>
<tr>
<td>Beam pattern 53cm line</td>
<td>≈1.5° at 117kHz</td>
</tr>
<tr>
<td></td>
<td>≈60° at 117kHz</td>
</tr>
<tr>
<td>Efficiency</td>
<td>&gt;40%</td>
</tr>
<tr>
<td>Input power &lt;5% duty cycle</td>
<td>1500W</td>
</tr>
<tr>
<td>Operating depth</td>
<td>unlimited</td>
</tr>
<tr>
<td>Weight</td>
<td>4.3kg</td>
</tr>
<tr>
<td>Housing</td>
<td>aluminium</td>
</tr>
</tbody>
</table>

Figure 12.24: ITC sidescan transducer array specifications
12.7.3. Operation and Image Interpretation

The array is towed through the water at a speed between 5kt and 15kt depending on the maximum operational range and the azimuth beamwidth, and the returned echo intensity is plotted as a function of round trip time on the range axis, and synchronised to the tow ship velocity on the cross range axis.

Figure 12.25: Side-scan sonar image of the wreck of the Port Hunter

The black line running vertically through the centre of the image is the bang pulse

The bare white region on either side of this is the slant range from the array to the intersection of the lower edge of the beam with the sea bottom with clutter in this region due to reflecting objects in the beam such as fish, krill etc.

The rest of the image is determined by the reflectivity properties of the sea bottom.

Figure 12.26: The Port Hunter
12.7.4. Shadowing

Because of the low grazing angles and large relative target heights, shadowing is often a pronounced feature of the images.

It can be used to generate an estimate of the height of the target above the sea floor.

![Figure 12.27: Shadowing may be useful to identify the height of the target as can be seen in this sidescan image of a wreck](image)

12.7.5. Signal Processing

The standard matched-filtering principles developed for radar are applied to sonar systems to ensure that the maximum SNR is achieved.

Most side-scan systems are real beam in that their cross range resolution is a function of range \( x = R \theta \). However digital techniques can be applied to correct for phase front curvature. This is known as focussing, and it can be used to achieve a fairly constant linear beamwidth with range. The process is similar to that used for Synthetic Aperture Radar (SAR).

Beam scanning techniques using phase shifters in the arrays can be used to spotlight particular areas and pseudo 3D images can be created if views are made from more than one perspective.

Simultaneous multi-frequency operation eg. 100kHz and 600kHz is possible for high resolution short range operation and lower resolution long range operation.
Note the fine range and angular resolution that can be achieved due to the short operational wavelength

### 12.7.6. Performance Evaluation (ICT-5202 Transducer)

What is the smallest target that the sidescan sonar based on the ITC-5202 transducer can see at a range of 500m?

It operates at a resonant frequency \( f_r = 117 \text{kHz} \) which equates to a wavelength \( \lambda = 13\text{mm} \) if the velocity of sound is taken to be \( c = 1522\text{m/s} \)

For a useable frequency range (Bandwidth \( B \)) from 111 to 126kHz, the quality factor \( Q \) of the transducer is calculated to be

\[
Q = \frac{f_r}{f_u - f_l} = \frac{117}{126 - 111} = 7.8.
\]

(12.33)

The rise time of any pulse generated by the transducer is related to the resonant frequency and the quality factor

\[
\tau_{\text{rise}} = \frac{Q}{f_r} = \frac{7.8}{117 \times 10^3} = 66.7 \mu s.
\]

(12.34)

The minimum pulse-width must be at least twice the rise time if the pulse it to reach its peak value. As a rule of thumb, for a “rectangular” pulse, the total pulse-width is \( 5\tau_{\text{rise}} = 333\mu s \). This equates to a pulse width, and an effective range resolution of

\[
\delta R = \frac{c}{2} \frac{\tau}{2} = \frac{1522 \times 333 \times 10^{-6}}{2} = 0.25m.
\]

(12.35)

The cross range resolution at 500m (for the given beamwidth of 1.5°) is the product of the beamwidth in radians and the range

\[
\delta XR = R\theta = 500 \times \frac{1.5}{57.3} = 13.1m.
\]

(12.36)
To operate out to a maximum unambiguous range of 500m, the maximum pulse repetition frequency PRF is

\[ PRF_{\text{max}} = \frac{c}{2R_{\text{max}}} = \frac{1522}{2 \times 500} = 1.52 \text{Hz}. \]  
(12.37)

The transmitter power is limited to a maximum of 1500W for a duty cycle, \( DC \), of less than 5%. The duty cycle in this case is

\[ DC = 100 \times PRF \tau = 100 \times 1.52 \times 333 \times 10^{-6} = 0.05%, \]  
(12.38)

which is much smaller than the limit, so the maximum power can be applied to the transmitter.

If the transducer was omnidirectional, then the power density at a range of 1m would be the product of the electrical power, \( P_{\text{elec}} \), and the conversion efficiency, \( \eta \), divided by the surface area of a sphere with a radius of 1m

\[ I_{\text{iso}} = \frac{P_{\text{elec}} \eta}{4\pi} = \frac{1500 \times 0.4}{4\pi} = 47.7 \text{W/m}^2. \]  
(12.39)

The antenna gain, known as the Directivity Index, \( DI \), which is defined in terms of the power with respect to an isotropic radiator can be calculated from the elevation and azimuth beam widths (in radians)

\[ G = \frac{4\pi}{\theta \phi} = \frac{4\pi \times 57.3^2}{1.5 \times 60} = 458.4. \]  
(12.40)

The actual power density in the direction of the peak gain is the product of the gain and the isotropic value

\[ I = I_{\text{iso}} G = 47.7 \times 458.4 = 21864 \text{W/m}^2. \]  
(12.41)

The sound pressure level (SPL) or \( S \) is generally given in dB relative to 1\( \mu \)Pa at a range of 1m. This can be calculated from the power density and the acoustic impedance \( Z \) of the water.

The acoustic impedance of water is the product of the density and the velocity

\[ Z = \rho_c c = 1026.4 \times 1522 = 1.56 \times 10^6 \text{kg/m}^2 \text{sec}. \]  
(12.42)

The relationship between the acoustic pressure, \( P \), in Pascals, the power density, \( I \), in W/m\(^2\) and the impedance, \( Z \), is

\[ P^2 = IZ. \]  
(12.43)
This can be re-written for the acoustic pressure in \( \mu \text{Pa} \) for the sound pressure level \( S \) as follows

\[ S = (10^6 P)^2 \times 10^{12} IZ. \]  

(12.44)

This is generally written in dB form

\[
10 \log_{10} S = 20 \log_{10} (10^6 P) = 10 \log_{10} 10^{12} + 10 \log_{10} (IZ) \\
S_{dB} = 120 + 10 \log_{10} (21864 \times 1.56 \times 10^6) = 120 + 105.3 = 225.3 \text{dB}
\]  

(12.45)

If the transmitting voltage response (TVR) shown in the transducer specification table is examined. It can be seen that at 117kHz, the response is 170dB rel to 1\( \mu \text{Pa/Volt} \) at 1meter.

The electrical power input \( P_{\text{elec}} \) is related to the RMS voltage \( V \) and the transducer conductance \( G \)

\[ P_{\text{elec}} = V^2 G. \]  

(12.46)

For a conductance \( G = 5.5k \mu \text{mho} \) from the transducer specification table and a power \( P_{\text{elec}} = 1500 \text{W} \)

\[
V = \sqrt{\frac{P_{\text{elec}}}{G}} = \sqrt{\frac{1500}{5.5 \times 10^{-3}}} = 522 V_{\text{rms}}.
\]  

(12.47)

The sound pressure level \( S_{dB} \) for a RMS voltage of 522V applied to the transducer is

\[ S_{dB} = 170 + 20 \log_{10} (V) = 170 + 20 \log_{10} (522) = 224.3 \text{dB}. \]  

(12.48)

This is as expected, because it should equal the sound pressure level calculated from the maximum input power. The 1dB difference could be due to some of the assumptions made, and difficulties with reading the graphs accurately.

As the signal propagates through the water, the sound pressure level reduces because the wave is expanding on a spherical wave-front and due to attenuation. The transmission loss in dB is \( H \) and is determined as

\[ H = 20 \log_{10} \frac{r_2}{r_1} + \alpha_{dB} (r_2 - r_1). \]  

(12.49)

Because the sound pressure is determined relative to the level existing at one meter from the effective centre of the sound source, the equation can be rewritten for this reference distance as follows

\[ H = 20 \log_{10} r + \alpha_{dB} r. \]  

(12.50)
The attenuation in dB/m is given by the following formula where \( f \) is the frequency of the sound in kHz.

\[
\alpha_{dB} = \frac{0.036 f^2}{f^2 + 3600} + 3.2 \times 10^{-7} f^2 = \frac{0.036 \times 117^2}{117^2 + 3600} + 3.2 \times 10^{-7} \times 117^2 = 0.0329 \text{dB/m}
\]

The target strength, \( T \), in dB is defined by ratio of the reflected sound pressure scattered by the target at a distance of one meter from the effective centre of the scattered sound to the incident sound pressure on the target as follows

\[
T = 20 \log_{10} \frac{P_r}{P_i}.
\] (12.51)

This target strength is determined by its size, shape and the fraction of sound that is re-radiated.

If the scattering cross section is \( \sigma \) square meters, then \( T \) in dB is given by

\[
T = 10 \log_{10} \frac{\sigma}{4 \pi}.
\] (12.52)

With the radar case, a sphere with a radius, \( a \) much larger than the wavelength will have a cross section equal to the projected area and \( T \) can be rewritten to take that into account

\[
T = 10 \log_{10} \frac{\pi a^2}{4 \pi} = 10 \log_{10} \left( \frac{a}{2} \right)^2 = 20 \log_{10} \frac{a}{2}
\] (12.53)

The target strength of World War II submarines varied between about 10dB at the bow to 25dB broadside and 15dB at the stern.

For a spherical target, the echo sound pressure level \( E \) relative to 1\( \mu \)Pa at a range of 1m from the receiver is easily calculated as follows

\[
E_{dB} = S_{dB} - 2H + T ,
\] (12.54)

\[
E_{dB} = S_{dB} - 40 \log_{10} r - 2\alpha_{dB} r + 20 \log_{10} \frac{a}{2}.
\] (12.55)

If the target is the sea floor, then the cross section will be the product of the range resolution and the cross range resolution modified by a scaling factor to take into account the reflectivity \( \sigma' \) of the surface.

\[
T = 10 \log_{10} \frac{\sigma' A}{4 \pi} = 10 \log_{10} \frac{\sigma' \delta R \delta X R}{4 \pi} = 10 \log_{10} \frac{\sigma' \delta R \delta \rho_{\rho_{\theta_{\phi}}}}{4 \pi} ,
\] (12.56)
and the echo sound pressure level will be

\[ E_{db} = S_{db} - 40 \log_{10} r - 2 \alpha_{db} r + 10 \log_{10} \frac{\sigma^2 \delta R \theta_{az}}{4\pi} r . \]  

(12.57)

Substituting for \( S_{db} = -225.3 \text{dB} \),
\( \alpha_{db} = -0.0328 \text{ dB/m} \),
\( \sigma^2 = 0.1 \text{ m}^2/\text{m}^2 \),
\( \theta_{az} = 1.5^\circ (0.0262 \text{ rad}) \),
\( \delta R = 0.25 \text{m} \),

\[ E_{db} = 225.3 - 40 \log_{10} r - 0.0656r - 42.8 + 10 \log_{10} r . \]  

(12.57)

As with the radar range equation, detection can only take place if the signal exceeds the noise level by a specific margin. It will be assumed that the noise is white and the signal is sinusoidal so that the same graphs can be used.

The noise level at sea is mostly generated by wind and wave action on the surface. It is proportional to sea-state and inversely proportional to frequency. All the sources of noise are shown in Figure 12.29.

**Table 12.2: Noise pressure as a function of sea state**

<table>
<thead>
<tr>
<th>Sea State</th>
<th>Wind Speed (knots)</th>
<th>Noise Pressure ( N_i ) (dB rel 1( \mu )Pa) Isotropic, 1Hz Bandwidth at 1kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>35</td>
<td>70</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>65</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>55</td>
</tr>
<tr>
<td>0.5</td>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>45</td>
</tr>
</tbody>
</table>

The frequency relationship to map the noise pressure level at 1kHz to the transducer frequency is

\[ N_f = N_i - 17 \log_{10} f_{\text{kHz}} \]  

(12.58)

For a sea state 3 and the transducer frequency of 117kHz

\[ N_f = 65 - 17 \log_{10} 117 = 30 \text{dB} \]  

(12.59)

The total noise pressure level in dB relative to 1\( \mu \)Pa must take into account the bandwidth \( B \) of the transducer (in Hz) and its directivity or gain \( G \) because the sensor is only looking at a small part of the hemisphere.

\[ L_N = N_f + 10 \log_{10} B - 10 \log_{10} G \]  

(12.60)

\[ L_N = 30 + 10 \log_{10} (15 \times 10^3) - 10 \log_{10} (458.4) = 45 \text{dB} \]
A second source of noise called reverberation noise is equivalent to volumetric clutter in radar, and will not be considered here, neither will all the other sources of noise shown in the figure above, be considered.

The echo sound level for various perfectly reflecting spheres and the ground and the noise pressure are plotted in the figure below.
The sea bottom should be detectable at 500m as would a sphere with a diameter of 1m.

Detection of the 0.1m sphere would be marginal as though the SNR is positive, the 13dB margin required for a $P_d = 0.9$ and a $P_{fa} = 10^{-6}$ is not there.

The actual voltage output by the transducer is determined from the transducer specifications. The open circuit receiving response at 117kHz is $-180\text{dB rel 1V/\mu Pa}$. For a signal pressure of 70dB (the sea-floor return at 500m), the output is

$$20\log_{10}(V) = 70 - 180 = -110\text{dB}$$

This can be converted to a voltage

$$V = 10^{-110/20} = 3.16\mu V$$

Which, in this case, is very small, and so receiver noise would be a consideration when the actual detection characteristics of the system were being considered.

### 12.8. Other Applications

Sonar technology has also become a valuable resource in volcanology studies. In 2003, NOAA began a survey of submarine volcanoes along the Mariana Arc north of Guam. Seafloor mapping technology has allowed the group to successfully monitor and track the activity of these underwater volcanoes. The figure below shows the Nikko Volcano, imaged using a multibeam sonar system. The history of the volcano...
can be deduced from the seafloor map and current activity, such as the thermal plumes and lava flows, can be monitored by geologists.

Figure 12.31: Sonar image of the Nikko submarine volcano

12.9. Phased Array Image Gallery

Figure 12.32: A few interesting sidescan sonar images
12.10. Doppler Beam-Sharpening

As an introduction to synthetic aperture processing, it is useful to consider a similar, but more basic signal processing technique called Doppler beam-sharpening. It uses the different Doppler shifts across the antenna beam footprint to narrow the effective beamwidth.

![Figure 12.33: Process of Doppler Beam Sharpening](image)

The resolution of a real aperture imaging radar is determined by the range resolution and the azimuth beamwidth, as shown in the figure. If the radar is moving at a constant velocity, then the dotted lines in the figure show lines of constant Doppler shift (isodop lines).

If the beam is offset from the direction of travel, then the isodop lines get closer together, and it becomes easier to isolate smaller sections of the beam by their relative Doppler shifts.

For lines of constant velocity (isovel lines) separated by $\Delta V$ m/s, the angular spacing will be

$$\theta_N = \arccos \left( 1 - \frac{N \Delta V}{V_t} \right)$$ (12.63)

For $V_t = 250$ m/s and $\Delta V = 1.25$ m/s the isovel lines will be at the following angles which correspond to those in the figure.
The Doppler shift of a target viewed from an aircraft travelling at $V_t$ (m/s), at an angle $\theta$ (deg) to the direction of travel is

$$f_d = \frac{2V_t \cos \theta}{\lambda} \quad (12.64)$$

If the radar operates at a frequency of 10GHz, then these isovel lines become isodop lines with a separation of 83Hz.

One of the limitations to the amount of sharpening that can occur is the observation time. To obtain a resolution of 83Hz in the received Doppler requires that the target be observed for $\tau = \frac{1}{\beta} = \frac{1}{83} = 12$ms. However, at a speed of 250m/s, the target would have moved 3m during this period, and that might be bigger than the range resolution. This is referred to as range walk, and is accommodated in both unfocussed and focussed SAR.

For a fixed transmission frequency, there as therefore a trade off that must be made between the range resolution, and the amount of sharpening that occurs.

An alternative is to increase the transmit frequency as this will increase the Doppler shift. For example, if the radar operates at 94GHz, then the separation in the isodop lines increases to 800Hz, and the observation time is reduced to 1.2ms during which time the aircraft only moves 0.3m.

A reflectivity image is constructed by physically scanning the antenna beam to the one side (or both) of the direction of travel, with the pixel size determined by the range resolution and the sharpened angular resolution.

The following figure shows a comparison between a real beam millimetre wave image, and one that has been Doppler beam-sharpened. An examination of point targets in the two images show that even though the cross range resolution has only been improved by a factor of 2, the image quality is much improved.
12.11. Operational Principles of Synthetic Aperture

The term “synthetic aperture” refers to the distance that the sensor travels during the time that the reflectivity data are collected from a single point. Energy from each scatterer is made to arrive in phase at the output of the processor for all of the samples to realise the narrow beamwidth. The process of synthesis relies on the fact that the wide antenna beamwidth can be subdivided into narrow slivers using the differences in the Doppler shifts across the beam as shown in the figure below.

![Figure 12.35: Synthetic aperture generation](image)

The process to determine the radiation pattern is similar to that used for the fixed array, with the primary difference being that the signal received by each element is due only to the received power scattered by target P from one transmitter element. This results in a slightly different radiation pattern for SAR. The beamwidth is narrower, but the sidelobes are higher than that for the equivalent phased array.
For, $L_c$, the synthetic array length,

$$G_{SAR}(\theta) = \frac{\sin^2 \left( \frac{\pi L_c}{\lambda} \sin \theta \right)}{\left( \frac{\pi L_c}{\lambda} \sin \theta \right)^2}.$$  
(12.65)

The half power beamwidth can be found by solving for $G_{SAR}(\theta) = 0.5$ and solving graphically (or using Newton) to show that

$$\frac{\pi L_c}{\lambda} \sin \theta \approx \frac{0.886\pi}{2} = 1.39.$$  
(12.66)

12.12. Range and Cross Range Resolution

In most SAR applications, the good range resolution is achieved by pulse compression and the cross-range resolution is obtained using synthetic aperture processing. It is useful to plot the radar response to a point scatterer on the range-azimuth plane as the various processes which determine the overall resolution can be illustrated graphically.

Figure 12.36: Generation of a synthetic array

Figure 12.37: Range-azimuth response to a point scatterer shows range walk and range curvature (a) before pulse compression and (b) after pulse compression
12.12.1. Unfocussed SAR

In unfocussed SAR, all aircraft motion that deviates from a straight line is compensated for. In addition, for a forward squinted radar, the reduction in range induced by the radar velocity, called range-walk, is also removed prior to processing.

However, unfocussed SAR makes no attempt to correct for the relative phase shift to a point target across the aperture which is known as range curvature. The limiting condition is, therefore, the point where the phase error reaches $\lambda/4$ as determined in the following construction

$$\frac{\lambda}{4} = \frac{L_{\text{max}}}{2} \sin(\theta/2)$$  \hspace{1cm} (12.67)

for $\sin(\theta/2) \approx \frac{L_{\text{max}}}{2R}$

$$\frac{\lambda}{4} \approx \frac{L_{\text{max}}^2}{4R}$$ \hspace{1cm} (12.68)

$$L_{\text{max}} = \sqrt{R\lambda}$$ \hspace{1cm} (12.69)

A second limiting condition is that the beamwidth is sufficiently wide to illuminate the target at point P. $L_{\text{max}} \leq R\theta_{3dB}$
The beamwidth is obtained by equating $G_{\text{SAR}}(\theta) = 0.5$ as before

$$\frac{\pi L_e}{\lambda} \sin \theta \approx \frac{0.886\pi}{2} = 1.39$$

(12.70)

$$\sin \theta = \frac{0.886\pi\lambda}{2\pi L_e}$$

(12.71)

The cross-range resolution $\delta_{cr} = R\theta = R\sin \theta$ for small angles

$$\delta_{cr} = \frac{R0.886\pi\lambda}{2\pi L_e}$$

(12.72)

Substituting $L_e = L_{\text{max}} = \sqrt{R\lambda}$ and simplifying

$$\delta_{cr} \approx 0.5\sqrt{R\lambda}$$

(12.73)

12.12.2. Focussed SAR

In the focussed SAR case both range-walk and range curvature are removed so that all of the returns from a point scatterer are concentrated at a single range. This allows the signal to be correlated across the azimuth plane to produce the required cross-range resolution.

![Figure 12.40: Effect of range-curvature removal on a range-compressed point scatterer signature](image)

This cross-range resolution can be determined from a Doppler perspective by considering the following:

A point scatterer enters the forward edge of the beam as shown in the figure below. It will have Doppler frequency

$$f_d = \frac{2v_e}{c} = \frac{2v}{\lambda} \cos(\theta_{\text{inc}}/2).$$

(12.74)
For small beamwidths, the Doppler frequency decreases linearly to 0 and then increases again and the angle to the target $\theta \approx \frac{v t}{R}$ as a function of time.

The Doppler frequency as a function of time will then be

$$f_d(t) = \frac{2v_t}{\lambda} = \frac{2v}{\lambda} \cos \left[ \frac{vt}{R} \right]. \quad (12.75)$$

Taking the derivative to obtain the rate of change of Doppler frequency, or the Doppler slope

$$\frac{df_d}{dt} = \frac{2v}{\lambda} \frac{v}{R} \sin \left[ \frac{vt}{R} \right]. \quad (12.76)$$

equating at $t = 0$,

$$\frac{df_d}{dt} = \frac{2v^2}{R\lambda}. \quad (12.77)$$

The total Doppler shift over time $T_d = $ time within the beam for $\theta = -\theta_{3dB}/2$ to $+\theta_{3dB}/2$, assuming a linear change in frequency

$$\Delta f_d = \frac{2v^2}{R\lambda} T_d. \quad (12.78)$$

By analogy to the linear FM range resolution, the signal can be passed through a matched filter to give a spectral resolution $\delta f = 1/T_d$. 

---

**Figure 12.41: Antenna beamwidth limitations to maximum aperture**
The cross range resolution is then the optimised cross range resolution of the real beam \( \delta_b = L_c \) scaled by the ratio of the spectral resolution to the whole Doppler shift

\[
\delta_{cr} = \delta_b \cdot \frac{\Delta f}{\Delta f_d} = L_c \frac{\Delta f}{\Delta f_d}.
\]  
(12.79)

Substituting (12.78) into (12.79)

\[
\delta_{cr} = L_c \frac{R \lambda}{2v^2T_d} \cdot \frac{1}{T_d} = L_c \frac{R \lambda}{2v^2T_d^2}.
\]  
(12.80)

But \( L_c = vT_d = R\theta_b \frac{R\lambda}{D} \) where \( D \) = antenna aperture, so the cross range resolution becomes

\[
\delta_{cr} = \frac{R \lambda}{2L_c} = \frac{R \lambda}{2} \frac{D}{R \lambda} = \frac{D}{2}.
\]  
(12.81)

The cross range resolution for focussed SAR is independent of the range \( R \)

**12.12.3. Resolution Comparison**

For a radar with a real aperture of 150mm and a frequency of 77GHz the following figure shows the relationship between the cross-range resolution of a real aperture antenna, unfocussed SAR and focussed SAR.

![Resolution comparison: real aperture, unfocussed SAR and focussed SAR](image)

**Figure 12.42: Resolution comparison: real aperture, unfocussed SAR and focussed SAR**
12.13. Example: Synthetic Aperture Sonar

Three targets are placed with cross range separations of 20mm and 35mm at a range of 2m. Coordinates [20,2000], [40,2000], [75,2000] mm

The antenna aperture \(a = 10\)mm produces a 3dB beamwidth of 49°

As the sonar moves past the targets the received phase of the echoes is determined and shown in the following figure.

![Figure 12.43: Received phase from three targets](image)

The focussed returns are generated by subtracting a reference phase profile generated by a fixed target at [0,2000] making the phase shift with radar position is now almost linear.

![Figure 12.44: Received phase from three targets after focussing](image)
The received signal is reconstructed by taking the cosine of the individual phases and summing the results.

**Figure 12.45:** Received signal from the three targets

The individual targets are resolved by examining the spectrum of the received signal.

**Figure 12.46:** Resolved target cross range positions for $d = 10\text{mm}$

Increasing the antenna aperture to $d = 30\text{mm}$ decreases the cross range resolution, and the two closest targets are no longer resolvable.
Figure 12.47: Resolved target cross range positions for $d = 30\text{mm}$

% Synthetic Aperture Sonar Model
% sar01.m

degrad = pi/180; % degrees to radians
c = 340; % speed of sound (m/s)
f = 40e+03; % operational frequency (Hz)
d = 10.0e-03; % antenna aperture (m)
lam = c/f; % wavelength (m)

% generate three targets at the same range separated by
% a fraction of the beamwidth
x0 = 0;
y0 = 2.0; % reference target to determine phase shift for focus
x1 = 20.0e-03;
y1 = 2.0;
x2 = 40.0e-03;
y2 = 2.0;
x3 = 75.0e-03;
y3 = 2.0;

% assume a rectangular aperture and determine the 3dB beamwidth
thet3 = asin(0.887*lam/d);

% generate a moving radar system along the x axis
lmax=2*y1*sin(thet3/2);
dx = lmax/511;
xrad = -lmax/2:dx:lmax/2;
yrad = zeros(size(xrad));

% calculate the range to each of the targets
dx1 = xrad-x1;
dy1 = yrad-y1;
dx2 = xrad-x2;
dy2 = yrad-y2;
dx3 = xrad-x3;
dy3 = yrad-y3;

[az1,r1]=cart2pol(dx1,dy1);
[az2,r2]=cart2pol(dx2,dy2);
\[ [az3, r3] = \text{cart2pol}(dx3, dy3); \]

% calculate the range to the reference target for focusing
\[ dx0 = xrad - x0; \]
\[ dy0 = yrad - y0; \]
\[ [az0, r0] = \text{cart2pol}(dx0, dy0); \]

% calculate the phase due to the round trip distance to each target
\[ ph1 = 4\pi f \cdot r1 / c; \]
\[ ph2 = 4\pi f \cdot r2 / c; \]
\[ ph3 = 4\pi f \cdot r3 / c; \]

\[ ph0 = 4\pi f \cdot r0 / c; \] % reference phase

plot(xrad, ph1, xrad, ph2, xrad, ph3) 
grid 
title('Received Phase Before Focusing') 
xlabel('Radar Position (m)') 
ylabel('Received Phase (rad)') 
pause

% subtract the reference phase to focus
\[ dph1 = ph1 - ph0; \]
\[ dph2 = ph2 - ph0; \]
\[ dph3 = ph3 - ph0; \]

plot(xrad, dph1, xrad, dph2, xrad, dph3) 
grid 
title('Received Phase After Focusing') 
xlabel('Radar Position (m)') 
ylabel('Received Phase (rad)') 
pause

% generate the time domain signal from the received phases
\[ sig1 = \cos(dph1); \]
\[ sig2 = \cos(dph2); \]
\[ sig3 = \cos(dph3); \]
\[ sig = sig1 + sig2 + sig3; \]

plot(xrad, sig1, xrad, sig2, xrad, sig3) 
grid 
title('Received Signal') 
xlabel('Radar Position (m)') 
ylabel('Received Signal Amplitude (rad)') 
pause

% extract the target returns by looking at the received spectrum
\[ sigf = \text{fft}(sig); \]
\[ db = 20 \log_{10}(|\text{abs}(sigf(1:255))|); \]

plot(db) 
grid 
title('Received Signal Spectrum') 
xlabel('Frequency') 
ylabel('Amplitude (dB)') 
axis([0, 50, 0, 50])

12.14.1. Perspective of a Radar Image

Figure 12.48: Perspective in a radar is different to that from an aerial photograph because in a radar two objects coincide if they are at the same range, whereas in an optical system, they appear to coincide if they're at the same angle.

12.14.2. Image Distortion

The most common forms of distortion that are suffered by radar images made from the air include the following:

- layover, when the range to the top of an object is less than the distance to its base
- foreshortening, when the near side of elevated objects appears steeper than it actually is
- stretching due to the differences between the ground and slant ranges
- shadowing, when a tall opaque object blocks the signal path behind it, and no returns are received.

Figure 12.49: Some image distortion effects
Stretching

Figure 12.50: The radar image is distorted and compressed by the non-linear mapping between the ground range and the slant range

Shadowing

As illustrated in an earlier figure, shadowing occurs mostly at low grazing angles when an object, or objects block the signal path to cast shadows which cannot be imaged by the radar.

Figure 12.51: For low grazing angle applications, buildings and mountains cast shadows which are not imaged by the radar. These appear as black regions in SAR images
12.14.3. Speckle

Because the measurement process is coherent, in some of the image pixels, the scatterers will add constructively to produce high intensity returns, and in some the summation will be destructive to produce dark returns.

Various filtering and averaging techniques are used to reduce these effects and to make the images easier to interpret.

![Constructive Interference](image1)

![Destructive Interference](image2)

**Example of Homogeneous Target**  
(being imaged by a radar sensor)

![Example of Homogeneous Target](image3)

Figure 12.52: SAR Speckle and examples of some filter processes used to reduce its effect

12.15. Airborne SAR

Probably the best known of all the SAR systems designed for use in UAVs is the xxx installed in the Predator.

TESAR (Tactical Endurance Synthetic Aperture Radar) (also called the MAE UAV SAR) is a strip mapping SAR providing continuous 0.3 meter (1 foot) imagery. The focused imagery is formed on-board the Predator aircraft, compressed and sent to the
Predator Ground Control Station over a Ku band data link. The imagery is reformed and displayed in a scrolling manner on the SAR workstation displays. As the imagery is scrolling by, the operator has the ability to select 1k by 1k image patches (approximately 800m x 800m box at 15,000 AGL) for exploitation at the Predator DEMPC workstation. The imagery is also recorded continuously on DLT tapes.

There are 2 modes of operation. Mode 1 provides a non-centered strip map. That is the map center moves with respect to the aircraft motion. Mode 2 is the classic strip map mode. Mapping occurs over a predetermined scene center line, irrelevant of the aircraft motion.

The radar is designed to map while squinting up to ±45 degrees off the velocity vector. At ground speeds from 25-35 m/sec, the swath width is 800 meters. At speeds beyond 35 m/sec, the swath width decreases proportionally with the increase in ground speed.

Other SAR developments for both UAV and manned aircraft include those by Dornier and Sandia.
By the end of 2006 Sandia National Laboratories was flying the smallest SAR ever to be used for reconnaissance on near-model-airplane-sized unmanned aerial vehicles (UAVs). Weighing less than 10kg, the miniSAR is one-fourth the weight and one-tenth the volume of its predecessors currently flying on larger UAVs such as the General Atomics’ Predator. It is the latest design produced by Sandia based on more than 20 years of related research and development.

The new miniSAR will be able to take high-resolution (four-inch) images through bad weather, at night, and in dust storms out to a range of about 15km.
Figure 12.56: X-band SAR image of airport from Sandia labs

Figure 12.57: X-band SAR image of China Lake airfield (3m resolution)

Figure 12.58: SAR images of tanks and a ring of corner reflectors
12.16. Space Based SAR

To achieve good angular resolutions from real-aperture space-borne radars is impossible at lower frequencies because the size of the antenna becomes prohibitively large. As derived previously, the large synthetic aperture results in a cross range resolution independent of range, \( \delta_{cr} = D/2 \), where \( D \) is the antenna aperture.

The good range resolution with \( \delta_{r} = c/2\Delta f \) is achieved by transmitting a wide bandwidth chirp.

The primary advantage of space-borne SAR is because the trajectory of the satellite or shuttle is precisely known and stable, motion compensation is not required and exceptionally high quality images can be produced. However, an additional distortion due to the curvature of the earth that must be compensated for is shown in the figure below.

![Figure 12.59: Space Based SAR System](image)

![Figure 12.60: Geometric distortion due to the curvature of the earth](image)
12.16.1. Interferometry

Because SAR is concerned with the phase relationships between scatterers on the ground. If two similar images are produced using offset antennas, or on subsequent passes over the same area, the interference patterns can be used to determine the true height of the objects on the ground.

In addition to being useful for mapping ground features, this technology has a number of other uses:

- Local deformation of the earth’s crust as an early warning of earthquakes or volcanoes.
- Ground subsidence due to mining activities or excessive use of ground-water
12.16.2. SAR Image Gallery

C-band and X-band SAR systems have been flown by the shuttle over the last couple of years, and have produced the following incredible images.

Figure 12.62: The Namibian sand dunes (54×82km). In ultra arid regions such as this, the radar signal can penetrate the sand and produce images of sub-surface features such as abandoned stream courses.

Figure 12.63: Sydney
Figure 12.64: The Mississippi Delta (63×43km). The bright spots out at sea are oil rigs.
12.17. Magellan SAR Map of Venus

The radar sensor, Magellan’s sole scientific instrument, produced high resolution SAR images of more than 80% of the planet’s surface. It was also used as a radar altimeter and the high gain antenna was used to communicate with the earth.

Figure 12.65: Magellan SAR map of Venus and visible image (inset)

Figure 12.66: Sif Mons 2km high and 300km in diameter, 3D image produced by combining SAR and altimetry data

12.18. References

[13] Scientific American
[17] "X-band Synthetic Aperture Imagery, Sandia Labs,"