Chapter 17.
Tomography and 3D Imaging

17.1. Principle of Operation

The word tomography derives from the Greek word tomos meaning section, so the process of tomography involves the generation of narrow sections through an object. It is assumed that this is a non-invasive process which is performed using sensors outside the object of interest. In many applications sequences of measurements made in 2D are combined to produce a pseudo 3D image. The process is when applied to X-rays is referred to as Computerised Axial Tomography (CAT or CT).

![Figure 17.1: Computerised axial tomography (a) general principle and (b) as applied to a head](image)

Computed tomography only became feasible with the development of computer signal processing capabilities in the sixties, but many of the basic principles were developed many years before. In 1917, a mathematician, J. Radon showed that the distribution of material, or the material properties can be determined if the integral values along any number of lines passing through that layer, are known.

Such processing is common in medical applications where doctors require a 3D representation when planning invasive surgery.

Examples of this include:
• 3D Ultrasound
• 3D Magnetic Resonance Imaging (MRI)
• Positron Emission Tomography (PET)
• X-ray CT Scans

Ultrasound technology has been discussed in earlier lectures. It uses the different elasticity characteristics of the target to produce an image while MRI imaging relies on the varying amounts of Hydrogen (in general contained in water) and PET Imaging relies on the release of positrons from radioactive decay.

The best understood, and possibly the most common is X-ray CT imaging which relies on target density variations that affect the attenuation of high frequency EM waves.

17.2. CT Imaging

Between 1957 and 1963, a South African physicist, A. Cormick, independently developed a method of calculating radiation absorption distributions in the human body, based on transmission measurements. He postulated that that it would be possible to display very small differences in the absorption through the body, but never applied his research.

In the early 1970s CAT scanning (also called computerised tomography, or CT, scanning) was conceived, yet again, by William Oldendorf and developed by Godfrey Hounsfield. Although often employing contrast media to enhance the quality of the images obtained, this test is barely invasive.

The most basic CT measurement developed in the 1970s, involves the rotation and displacement a collimated X-ray source (pencil beam) around the patient as shown in the figure below.

![Figure 17.2: The simplest CT scan involves measuring the intensity of a pencil beam of X-rays from many different angular positions](image)
To speed up the process, more modern CT scanners consist of an X-ray source that produces a fan-beam which penetrates the patient and impinges on a bank of detectors. This complete assembly rotates around a central core to produce a sequence of intensity measurements over 360°.

In either case, a complex image reconstruction algorithm solves these equations derived from this massive set of measurements to produce an image which represents the density of the material in cross section through the patient.

The generation of clear images is compromised by movements of the patient during the scanning process, and so faster scan methods were developed. Continuously rotating CT systems that could image a complete slice in less than one second were first introduced in 1987. However, these systems still constructed 3D images from individual “slices” through the patient.

In the 1990’s spiral scanners with multiple slice capabilities were introduced to produce even faster 3D high resolution images. The state of the art is a scan time of less than 0.5s with a slice thickness of between 0.5 and 1mm.

Figure 17.3: CT scanner schematic
It allows visualisation of extremely thin sections of the brain, skull, spinal cord, and spine (as well as other parts of the body) in two dimensions and with enough clear distinction between black, grey, and white areas of the image to allow pathological diagnosis in many cases.

17.2.1. Image Reconstruction

As shown in Figure 17.2, measurements are made of the intensity of the X-ray beam and converted to a set of attenuation measurements. These are known as the “Radon Transform” of the image. An inverse transformation must be then carried out to determine the distribution of attenuations for each pixel \( \mu(x, y) \) element through the target.

The easiest process to understand is one in which there are \( N^2 \) unknowns in an \( N \times N \) matrix of pixels. If sufficient independent measurements are made, then it is possible to solve for all of the unknowns. In the simplest case of a target with only four elements, two measurements, from two projections will yield a system of four equations and four unknowns which can easily be solved. The extension to a \( 3 \times 3 \) matrix with nine unknowns can also be solved easily using the twelve equations as defined schematically in the figure.
This process was used by early CT scanners in which the number of elements was limited. However, with the requirement for finer resolutions and more pixels, the computational overheads became unacceptably high.

In modern CT scanners, the convolution-backprojection procedure is usually applied. It starts with a matrix loaded with zeros, and as each measurement is made, the projection value is added to all of the elements in the array along the direction of measurement. However, because each component does not contribute only to the value at the desired point, but to the whole image, it is clear that an unsharp image will result.

To minimise the magnitude of this unsharpening, each projection is convolved with a convolution kernel prior to backprojection. This convolution kernel represents a high pass filter.

Because convolution in the spatial domain is equivalent to multiplication in the frequency domain, it is possible to perform this convolution-backprojection process in that domain.

Figure 17.6: Image reconstruction by backprojection shows that convolution with a high pass kernel is required to ensure a sharp image
17.2.2. What is displayed in CT images

As explained earlier, the CT computes the spatial distribution of the linear attenuation coefficient $\mu(x,y)$. However, because $\mu$ is strongly dependent on the energy of the X-ray photons, it would make a direct comparison between images made with different systems impossible. Therefore, the image displayed as an attenuation relative to that of water. These are referred to as CT units, or sometimes Hounsfield Units (HU) in honour of the inventor.

$$CT_{\text{units}} = \frac{\mu(T) - \mu_{\text{water}}}{\mu_{\text{water}}} \times 1000$$  \hspace{2cm} (21.1)

On this scale, water, and consequently any water equivalent tissue with $\mu(T) = \mu_{\text{water}}$ has a value of 0 HU, by definition. Air corresponds to a value of -1000 HU because $\mu_{\text{air}}$ is close to zero. Bone and other calcifications with high atomic numbers and high density offer increased attenuation and therefore have higher CT values – typically up to 2000.

Most medical scanners cater for a range of CT values from -1024 to +3071, a total of 4096 values, as these can be represented by a 12bit number. Because, neither the imaging software, nor the eye can discern 4096 different shades of grey, CT ranges of interest for a specific imaging task, are windowed and expanded to fill all shades from black to white.

![CT values for normalised attenuation of different materials](image)

The contrast of a CT image is far superior to that of a conventional X-ray image, not because of the higher powers used, but because of the differences in the way that the images are generated.

In the figure below, a comparison is made between a CT image and an X-ray image. For the former a contrast between two arbitrary adjacent pixels is shown to be 28HU which corresponds to 50% in this scaling. The X-ray intensity will be determined by the sum of the attenuations along the whole path through which the beam travels. It can be seen in this example that the contrast between two adjacent paths through the cranium is only 0.23%
17.2.3. Two Dimensional Displays

CT Scanning is primarily limited to the transverse plane because of the mechanics of the scanning system. All of the other image planes are synthesized from the volume image constructed from these transverse slices.
17.2.4. Three Dimensional Displays

3D displays represent a scan volume in a single image which has generally been manipulated to enhance a specific characteristic. This can only be done successfully when one high-contrast structure (like the skeleton) is to be displayed. Shaded surface displays (SSD), maximum intensity projections (MIP) and volume rendering (VR) or perspective volume renderings (pVR) are some of the common manipulations.

Figure 17.10: Different 3D display methods (a) SSD for skeletal structures, (b) MIP for CT angiography, (c) VR for abdominal structures and (d) pVR for a virtual colonoscopy
17.3. MRI-Magnetic Resonance Imaging

MRI imaging uses the principle of Nuclear magnetic Resonance (NMR) and an assembly of powerful magnets to produce images generated by various elements within the patient. It is most commonly used to image hydrogen as the human body consists predominantly of that element.

17.3.1. Nuclear Magnetic Resonance (NMR)

Atomic nuclei have an angular momentum arising from their inherent property of rotation, or spin. Since the nuclei are electrically charged, the spin corresponds to a current flowing around the spin axis which in turn generates a small magnetic field.

![Diagram of magnetic field and spinning nuclei](image)

Each nucleus with an odd number of nucleons, and hence a non-zero net spin therefore has a magnetic moment, or dipole, associated with it. In general the orientation of the dipoles is random, but if they are placed in a magnetic field, they will become aligned to it.

Protons, or hydrogen nuclei, with a spin of $\frac{1}{2}$ can align parallel to the field or antiparallel. The two orientations have slightly different energy (Zeeman splitting) with the spin up state (parallel) having the lower energy. Their effects almost cancel, but the lower state has a slight excess that is exploited by the MRI.

![Diagram of aligned hydrogen atoms](image)

When a magnetic field is applied, the whole population of nuclei has a bulk magnetisation vector $M$ aligned to it. This is defined as the $z$-direction and by applying a small rotating magnetic field in the $x$-$y$ plane, the nuclei can be tipped away from the $z$-direction. The field rotation rate is tuned to the natural precession frequency of the nuclei (hence magnetic resonance). For Hydrogen nuclei in a magnetic field of 1Tesla ($10^4$ Gauss), the Larmor relationship which defines resonant frequency as a function of the applied field predicts the resonance to be 42.57MHz.
Other materials will have different resonant frequencies in the same field ($^{31}\text{P}$) has a resonance at 17.24MHz and ($^{23}\text{Na}$) at 11.26MHz.

It should be possible to “tune in” to any specific species to observe their response in isolation. However, for imaging purposes, only Hydrogen is used in practice because its concentration is high and it is more sensitive.

As the power in the rotating field pulse is increased, the bulk magnetisation vector $M$ will continue to rotate away from the direction of the static magnetic vector until it is orthogonal to it, rotating in the x-y plane only. When the pulse ends, the magnetisation vector continues to rotate for a time and in so doing, generates a small EMF in the field coils.

Interactions between adjacent nuclei that begin to spin at slightly different rates results in a loss of energy and causes the total spin to decay until it is once again aligned with the static field.

Figure 17.13: Principles of nuclear magnetic resonance showing (a) the unexcited magnetisation vector aligned with the external field (b) result of the application of an RF pulse to rotate the magnetisation vector (c)
17.3.2. Imaging Process

Creating Refined Anatomical Images

Within the metallic cocoon of an MRI scanner, the patient is surrounded by four electromagnetic coils and the components of a transceiver.

- **Scanner**: Uses electromagnets and radio signals to produce cross-sectional images.
- **X Coil**: Creates varying magnetic field from left to right across scanning tube.
- **Z Coil**: Creates varying magnetic field from head to toe within scanning tube.
- **Y Coil**: Sends radio signals to patient and receives signals from them.
- **Main Coil**: Surrounds patient with uniform magnetic field.
- **Patient**: Wears loose clothing; must empty pockets of metallic objects that could prove harmful if moved by magnetic force.

Figure 17.14: Schematic of a MRI machine
The imaging process exploits the time taken for the spin to decay, as this gives an indication of the proton, and hence water, density in the body.

As can be seen in the figure above, in addition to the fixed magnetic field generated by the main coil, three gradient coils can apply varying fields in the x, y and z directions. Manipulation of these fields is used to identify the spatial position of the hydrogen concentration.

As shown if the figure, if no gradient field is applied along the magnetic axis, then the Fourier transform of the decaying Larmor signal shows a single broadened spectrum.

If a magnetic field gradient is applied, then the Larmor spectrum will have characteristics of the variations in proton density profile in the direction of the gradient. By rotating the gradient field and producing proton density profiles from various angles, a computer can perform the inversion function to recreate in 2D the internal structure of the body. This process (using the projection method) is illustrated in the figure below.

This use of proton density makes it possible to produce images of tissues that are comparable, and in some cases superior, in resolution and contrast to those obtained with...
CT scanning. Moreover, since macroscopic movement affects NMR signals, the method can be adapted to measure blood flow.

Because the direction of imaging is determined by the magnetic gradient, it is possible to produce images in any plane; axial, coronal or sagittal.

![Reconstructed Image](image)

**Figure 17.16:** Image reconstruction using the projection method

17.4. MRI Images

![MRI Images](image)

**Figure 17.17:** MRI images of various parts of the body showing its capability to produce images of soft tissue
17.5. Functional MRI (fMRI) Investigations of Brain Function

fMRI investigations of brain function rely on the fact that oxygenated and deoxygenated haemoglobin molecules behave slightly differently in a magnetic field. MRI images can then be made to show the oxygen use in the brain using the Blood-Oxygen-Level-Dependent (BOLD) signal.

Images made using this technique, while the subject is observing a specific scene or thinking specific thoughts, are compared to those made by a control to determine where a specific brain function occurs.

The image below shows the difference between the subject looking at a face and looking at a blank screen.

![Functional MRI image of the areas of the brain used to recognise a face](image-url)
17.6. Positron Emission Tomography (PET)

Radioactive molecules made from radioactive isotopes with short half-lives such as $^{11}$C, $^{15}$O or $^{13}$N are injected into the body where radioactive decay takes place releasing a positron.

PET produces images of the body by detecting the gamma radiation – two photons each with an energy of 511keV that are created by the annihilation of a positron and an electron that occurs after this decay as shown in the figure below. These photons are emitted at very nearly 180° from one another.

![Figure 17.20: Emission of a gamma ray following annihilation of a positron and electron](image)

The gamma ray detector used in a PET scanner consists of an annulus of scintillation crystals each connected to a photo-multiplier tube as shown in the figure below. If a gamma ray strikes one of the crystals, it produces a photon of light that is converted to an electron and amplified by the tube to produce a measurable signal. A coincidence detector ensures that only pairs of hits that occur within about 25ns are recorded.

A coarse image is made of an axial slice through the body by combining the directions and relative time delays all of the measured events.

The scanner is moved and the process repeated until a 3D image has been built up.

![Figure 17.21: Schematic diagram of a PET scanner](image)
Depending on the type of molecule injected, PET can provide information on different biochemical functions.

For example, if the molecule that is radioactively tagged is glucose, then the PET scan will show an image of glucose metabolism, or how much energy the body is using in a specific area. This has been useful in studies of brain function.

PET scans do not have the same high resolution as that of fMRI shown in the previous section, but they are more sensitive in identifying physiological processes that use chemicals that do not have a good MRI signature. For example, abnormal bone growth that can indicate the presence of a tumour can be pinpointed using radioactive Phosphorus.

Figure 17.22: PET images showing brain function while words are processed in different ways

Figure 17.23: PET scan showing abnormal brain function of a METH user
17.7.3D Ultrasound Imaging

If the relative positions of the images made by a 2D-ultrasound scanner are known, then it is possible to build up a 3D image. In general, the transducer is still moved by hand as skilled Sonographers perform the function better and are less intimidating than mechanical positioners.

Some of the original work was done at UCSD on a suitably modified Acuson 128XP/10 with a C3 transducer. However, in the last few years a number of manufacturers have produced commercial versions of the 3D imager.
Figure 17.25: Process of converting a 2D ultrasound scan to a 3D image

Figure 17.26: Feet
Figure 17.27: Hands and faces
The scanning and processing speed of modern ultrasound machines is now so fast that manufacturers are offering movies. This is referred to as 4D ultrasound because it includes a time axis.
17.7.1. Ultrasonic Computed Tomography

Because the pulsed echo ultrasound process can only see tissue interfaces, an alternative is ultrasonic computed tomography which uses both the attenuation, and the propagation time to estimate both the attenuation coefficient, and the refractive index of the object.

17.8. 3D Sonar Imaging

Sophisticated 2D sonar arrays such as the one developed by Thomson Marconi Sonar (TMS) in Sydney can produce short range 3D images with voxol resolutions down to $1\times1\times1\text{mm}$. In this system a group of 3 uniformly spaced transmitters illuminates the target with high frequency (>1MHz) sound pulses.

A sparse phased array made up of 84 tiles each made up of a random pattern of 32 hydrophone receivers receives the echo.

![Figure 17.29: Complete non-scanned imaging sonar](image)
After a single pulse, the received amplitude and relative phase information from each of the 2688 receivers is processed to produce high resolution 3D images such as those shown below.

Figure 17.30: Individual 32 transducer receiver tile

Figure 17.31: 2D projection of a 3D image of a G clamp

Figure 17.32: 2D projection of a 3D image of a monkey wrench
17.9.3D Ground Penetrating Radar

A 2D-image slice is made by dragging a standard GPR across the ground and recording amplitude and phase information from each position and then synthesising the image using SAR techniques discussed earlier.

If a number of slices are produced side-by-side as with the axial tomography method, then a composite 3D image can be constructed.

This requires that the position of the GPR be well registered using differential GPS.

The following images made by Dean Goodman in Japan show a circular tomb and contents.

The use of GPR for archaeological surveys eliminates the damage caused by exploratory trenches that were employed in the past.

Figure 17.33: A pseudo 3D GPR image showing a 1100 year old circular burial moat adjacent to a more modern (600 year old) fence line.
Figure 17.34: 3D cutaway of a burial chamber shows the main chamber and a vertical shaft that leads to an offering below the burial

17.10. References

[2] Private Correspondence from Thomson Marconi Sonar (TMS).